

Do Personal Taxes Destroy Tax Shields?

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Abstract

In this note we discuss the relevance of personal taxes on tax shields. Tax Shields, TS, belong to shareholders and are affected by personal taxes on dividends that reduce the levered value of the firm. Debt holders also pay personal taxes but they do not affect Tax Shields.

We suggest that the traditional approach based upon Miller's Presidential Address in 1977 might underestimate the size of the effect of personal taxes either in the net tax shields and/or the net value associated to them. We also consider the irrelevance of personal taxes on interest received by debt holders on the size of tax shields earned by the firm on interest paid. We also conclude that Miller's approach shows some inconsistencies related to the behavior of Net Tax Shields when personal tax rate, Tps, and size of EBIT are considered.

We discuss the inconvenience of generalizing the situation of the arbitrage argument to the situation of real firms.

We propose a different expression for net tax shields and net value of those tax shields and suggest an approach for testing Miller's proposal with actual data.

Keywords: Debt Tax Shields, personal taxes, value of tax shields, optimal capital structure.

JEL Classification: E62, G12, G32, M21

Do Personal Taxes Destroy Tax Shields?

“Sire, to me it matters not whose son I am,
therefore I tell you that you are riding
without any clothes.”

Count Lucanor;
of the Fifty Pleasant Stories of Patronio,
Prince Don Juan Manuel, 1330-1335
first done into English
by James York, M. D., 1868

"But he isn't wearing anything at all!"

The Emperor's New Clothes
in *Fairy Tales Told for Children*-
Hans Christian Andersen 1835 – 1837

Introduction

In this note we discuss the relevance of personal taxes on tax shields. Tax Shields, TS, belong to shareholders and are affected by personal taxes on dividends that reduce the levered value of the firm. Debt holders also pay personal taxes but they do not affect Tax Shields.

We suggest that the traditional approach based upon Miller's Presidential Address in 1977 might underestimate the size of the effect of personal taxes either in the Net Tax Shields and/or the Net Value associated to them. We also consider the irrelevance of personal taxes on interest received by debt holders on the size of tax shields earned by the firm on interest paid. We discuss the inconvenience of generalizing the situation of the arbitrage argument proposed by Miller to the situation of real firms. We conclude that Miller's approach shows some inconsistencies related to the behavior of Net Tax Shields when personal tax rate, Tps, and size of EBIT are considered as variables for sensitivity analysis. We propose a different expression for net tax shields and net value of those tax shields and suggest an approach for testing Miller's proposal with actual data.

The work is divided in seven sections additional to the Introduction. In Section One we explain Miller's proposal and derive the same expression for Net Tax Shield or Net Tax Shield Gains. In Section Two we discuss the irrelevance of personal taxes on Tax Shields. In Section Three we derive an expression for Net Tax Shields without personal taxes on interest received by debt holders. In Section Four we show the consistencies and inconsistencies that have both approaches. In Section Five we estimate the Value of Tax Shields (VTS) After Personal Taxes under both approaches. In Section Six we propose a simplified procedure to examine with actual data the effect of personal taxes on Tax Shields for Miller and proposed approach. In Section Seven we have a summary and conclude.

Section One. Miller's proposal and its derivation

On the other hand, shareholders are subject of personal taxes that reduce the Cash Flow to Equity, CFE that affects the levered value of the firm.

Tax Shields, TS, are gained by firms and this cash flow goes directly to increase the cash flow to equity, CFE, and hence, belong to shareholders. In his famous "Debt and Taxes", Presidential Address, Miller (1977) extends the valuation model which corrects Modigliani and Miller (1958, 1963) and includes personal taxes. As a consequence, he suggests that shareholders are subject to personal taxes on dividends and these taxes reduce the TS and the Cash flow to Equity, CFE and affects the levered firm value. This happens in countries where dividends are taxed. Miller uses an arbitrage argument where the shareholder can replicate the leverage of the firm with third parties (*homemade leverage*). He suggests as well, that debt holders pay personal taxes on interest received and that affects TS. This simply means that both, shareholders and debt holders are the same and one. Moreover, Miller and his followers say that there is not tax shield from debt that makes firms to prefer using debt over equity to finance the investments in assets and this means that there is no Optimal Capital Structure, OCS. In short: TS are destroyed by personal taxes on dividends paid to shareholders and on interest paid to debt holders.

Since Miller (1977) popular corporate finance textbooks, (see for example, Benninga and Sarig (1997, p. 412), Berk and DeMarzo (2009, p. 473), Brealey, Myers, and Allen (2011, pp. 444-447) and Copeland y Weston (1992, pp. 451-454), among others) either derive equation (1) or simply, assume it as a given and say there is no saving in taxes for debt that makes firms to prefer the use of debt to finance investment in assets. The expression for the Net gain (value) of tax savings TS, is according to Miller (1977)

$$G = \left[1 - \frac{(1-T_c)(1-T_{ps})}{(1-T_{pb})} \right] B \quad (1)$$

where T_c is corporate tax rate, T_{pb} is personal tax rate on interest paid to debt holders and T_{ps} is the tax rate for personal shareholders on dividends and B is the market value of debt. According to Miller, the market value of a bond is contractual or book value of debt after personal taxes on interest. B is defined by

$$B = KdD(1-T_{pb})/r \quad (2)$$

Where K_d is the contractual cost of debt, D is the book value of debt and r is the risk free rate. This is an elegant assumption for something that the market does not provide. Market value of debt depends on the market rate of cost of debt, not on personal taxes on interest paid. The stock market does not value the after personal taxes cash flows from bonds and stocks as Miller proposes. See Damodaran A. (ca. 2006 and ND). Wansley (1998) referring to Sweeney, Warga, and Winters (1997) suggests that “finance theory almost always focuses on market value of debt, although in practice, most empirical work is limited to the use of book value of debt. This dichotomy is the result of the lack of generally available public quotes for bond issues. Most corporate debt is traded over the counter rather than listed on an exchange, and much of this debt trades infrequently.”

That elegance is reflected in the definition of Net Tax Shield Value or Gains, G , (eq (1)) as Miller calls it. With this definition equation (1) in terms of book value of debt, is

$$G = \left[1 - \frac{(1-T_c)(1-T_{ps})}{(1-T_{pb})} \right] \frac{K_d D (1-T_{pb})}{r} \quad (3a)$$

And Net TS is

$$Net\ TS\ (Miller) = \left[1 - \frac{(1-T_c)(1-T_{ps})}{(1-T_{pb})} \right] K_d D (1 - T_{pb}) \quad (3b)$$

$$Net\ TS\ (Miller) = [1 - T_{pb} - (1 - T_c)(1 - T_{ps})] K_d D \quad (3c)$$

After manipulating algebraically (1) and/or (3b) and when some conditions for T_c , T_{ps} and T_{pb} are met, the expression in brackets is zero and Miller and his followers say that the gain in Net Tax Savings and Net Tax Savings itself is zero and suggest the following expression proposed by Miller (1977):

$$(1-T_{pb}) = (1-T_c)(1-T_{ps}) \quad (4)$$

Equation (4) might be interpreted as one unit of tax shield after personal taxes on dividends is equal to a unit of after personal tax cost of debt.

How can we arrive to the expression for Net TS as posed by Miller? Simple. We apply what Miller tries to say: TS are destroyed by personal taxes in dividends paid to shareholders and interest paid to debt holders. With this in mind we have:

$$Net\ TS = TS - Dividends \times T_{ps} - Interest \times T_{pb} \quad (5a)$$

This simple and intuitive approach applies to perpetuities and finite cash flows (that is what occurs in reality). Using a familiar and widely known notation, we have

$$\text{Net TS} = K_d T_c D - (X - K_d D)(1 - T_c) T_p s - K_d D T_{pb} \quad (5b)$$

Grouping terms we have

$$\text{Net TS} = K_d D [T_c + (1 - T_c) T_p s - T_{pb}] - X(1 - T_c) T_p s \quad (5c)$$

Adding 1 and -1 to the terms in square brackets

$$\text{Net TS} = K_d D [(1 - T_{pb}) - 1 + T_c + (1 - T_c) T_p s] - X(1 - T_c) T_p s \quad (5d)$$

$$\text{Net TS} = K_d D [(1 - T_{pb}) - (1 - T_c) + (1 - T_c) T_p s] - X(1 - T_c) T_p s \quad (5e)$$

Grouping terms again,

$$\text{Net TS} = K_d D [(1 - T_{pb}) - (1 - T_c)(1 - T_p s)] - X(1 - T_c) T_p s \quad (5f)$$

Observe and compare equation (4) and the terms in square brackets in equation (5f). They have exactly the same elements. See equation (3b). However, observe as well the last term. This term seems as if personal taxes apply on Operating Income or EBIT. However, it does not; it is not the case. There are no personal taxes on EBIT as such, but corporate taxes. *This last term is part of personal taxes on Dividends.*

Section Two. Taxes on interest received are irrelevant for TS

But, do taxes paid by debt holders on interest affect the TS belonging to shareholders? We believe not. What debt holders pay in taxes on interest received might not affect TS gained by the company, which belongs to the shareholders. Personal taxes paid by the debt holder is outside of the firm and shareholder's cash flows. Therefore, personal taxes on interest received does not affect the value of the firm.

In a similar way, if labor costs increase, shareholders' cash flow will decline, but personal taxes paid by workers when receiving their wages do not affect shareholders' cash flows. Personal taxes on wage income are not relevant to shareholders. Similarly, interest expenses lower firm's and shareholders' cash flows, but shareholders are not concerned about personal taxes on debt holders' interest income. The reader should imagine how a cash flow (TS) belonging to the shareholders will be affected by the tax paid by banks or in general by debt holders. Perhaps, will the dividends be affected by taxation of the debt holders? Who can explain how taxes paid by a bank or the bondholder, (public debt) will destroy part of cash flows that belong to the shareholders?

This is a logic that comes from the approach of Miller (1977), that uses an arbitrage argument and concludes that if the shareholder acts simultaneously as a lender, then personal tax on interest received by debt holders destroys tax savings. It must be said that this is possible, but it is not a typical situation and should not be generalized.

This has been a position commonly and widely accepted for nearly the last 40 years, with no objection. In fact, this position dating back to 1977, from the Presidential Address by Miller (1977) already mentioned, has been derived algebraically or explicitly assumed by Graham (2003), Molnar and Nyborg (2011), Graham and Viswanathan (2015), and Lin and Flannery (2013). Niño et al, (2014), extend the formulas for debt tax savings in the line of thought of Miller (1977) to a firm with constant growth. They include the personal tax on interest, T_{pb} , as Miller and others do. They derive Miller's formula (1977), -which is not strictly derived in his 1977's paper-, accept his results and in addition they show how Miller could arrive to his formulation, but making, in our opinion, the same omissions as shown above with the "personal tax on EBIT". In addition, to generalize the approach of Miller, valid in the context of arbitration where the shareholder and the owner of the debt are the same person, distorts the amount of the Net TS, after personal taxes.

However, why personal taxes on dividends affect tax shields? Taxes on dividends affect tax shields because *both* belong to the shareholders. TS, belonging to the shareholders is not affected by taxes to the debt holders. It is out of the cycle of cash flows for the firm and for shareholders. It should be noted that the stock market when determining the value for a stock or a bond (debt), that value has no effects of personal taxes.

Section Three. Proposed approach

On the other hand, our approach is very simple and intuitive: TS earned by shareholders might be offset by taxes paid by shareholders on dividends. Hence, Net TS is TS minus personal taxes paid by shareholders because we assume that equity and debt holders are different people. It is the same as the one used in Section One to arrive to Miller's formulation. In this proposal we do not include personal taxes on interest received by debt holders, but as it can be seen in Section One, the analysis allows to introduce those taxes.

Hence, our proposal for TS after personal taxes, Net TS, is

$$\text{Net TS} = T_c K d D (1 - T_{ps}) + K d D T_{ps} - X(1 - T_c) T_{ps} \quad (6a)$$

All the terms has been defined previously.

The inflection or critical point is given by eq (6b) when TS and personal taxes are identical.

$$\text{Net TS} = KdDTc + KdD(1-Tc)Tps - X(1-Tc)Tps = 0 \quad (6b)$$

It is clear that this equation is not as elegant as (1) and (4), however, it is the correct one. Hence, when TS is completely destroyed it implies

$$V^{Un} = V^L \quad (7)$$

Where V^{Un} is the unlevered value of firm and V^L is the levered value of the firm.

Let us look at the cash flows involved in what is known as total cash flows, assuming that personal taxes on interest received are irrelevant for the firm and for the shareholders, unless they are the same and one person. We assume that shareholders and debt holders are different persons. We will analyze cash flows before and after personal taxes. We assume non growing cash flows at perpetuity to make easier the notation and the analysis and make it comparable with Miller's work.

Table 1a. Before personal tax cash flows*

Cash flows in perpetuity	Notation
EBIT after corporate tax = FCF	$X(1-Tc)$
Tax shields, TS	$KdDTc$
Total Cash Flow = FCF + TS	$X(1-Tc) + KdDTc$
Dividends, CFE	$(X-KdD)(1-Tc)$
Interest, CFD	KdD
Total Cash Flow = CFE + CFD	$(X-KdD)(1-Tc) + KdD$

*We define FCF = Free Cash Flow; TS = Tax savings; CFE = Cash flow to equity; CFD = Cash Flow to Debt. Source: Author's development.

The reader can verify that $FCF + TS = CFD + CFE$.¹

¹ In fact, the equations for total cash flows and values are identities.

Table 1b. After personal tax cash flows

Cash flows in perpetuity	Notation
EBIT after corporate tax, FCF	$X(1-T_c)$
Tax shields, TS	$KdDT_c$
Personal tax on Dividends	$Tps(X-KdD)(1-T_c)$
Net TS = TS - TpsDiv	$KdDT_c - Tps(X-KdD)(1-T_c)$
Total Cash Flow = FCF + Net TS	$X(1-T_c) + KdDT_c - Tps(X-KdD)(1-T_c)$
CFE = Dividends	$(X-KdD)(1-T_c)$
Personal tax on Dividends	$Tps(X-KdD)(1-T_c)$
CFE after personal tax	$(X-KdD)(1-T_c)(1-Tps)$
CFD	KdD
Total Cash flow	$(X-KdD)(1-T_c)(1-Tps) + KdD$

Source: Author's development.

The reader can verify that $FCF + \text{Net TS} = \text{CFE}(1-Tps) + \text{CFD}$

From Table 1b we conclude that Net TS is identical to (6a) and is consistent with the total cash flow equation.

From Miller (1977), Net TS is

$$\text{Net TS (Miller)} = TcKdD(1-Tps) + TpsKdD - TpbKdD \quad (8a)$$

When Tpb is dropped from the equation for the above mentioned reasons, we have

$$\text{Net TS (Miller without Tpb)} = TcKdD(1-Tps) + TpsKdD \quad (8b)$$

This is, the formulation by Miller (1977) for Net TS (eqs 8a) lacks part of the tax on dividends ($-X(1-T_c)Tps$) and has ($-TpbKdD$) in excess. Observe eq (8b). This equation says, that the reduction of TS by taxes on dividends is constant, given, T_c , r , D and Tps , independent from the size of dividends. And it should be clear and simple that personal taxes on dividends are what reduce Tax Shields from debt earned by the firm and owned by shareholders. Hence, the greater the dividends, the greater the reduction and the lower the Net TS.

Equation (6a) is the appropriate way to examine the effect of personal taxes on tax shields. This is, Ts minus personal Tax on dividends.

Section Four. Consistencies and Inconsistencies in Miller and Present Proposal

Now, we examine the behavior of Net TS with Miller's proposal and with the proposed approach.

Example 1

Assume $X = 20$, $D = 100$, $Tps = 10\%$, $Tc = 30\%$ and $r = 10\%$ and equation (8b) or TS (Miller without Tpb). For the given values, equation (8b) is 3.7. Table 2a show how Net TS (Miller) behaves when Tps and Tc change.

Table 2a. Size of Net TS, Eq (8b) (Miller's approach) for different Tc and Tps .

$Tc \backslash Tps$	20%	25%	30%	35%	40%
0%	2.00	2.50	3.00	3.50	4.00
5%	2.40	2.88	3.35	3.83	4.30
10%	2.80	3.25	3.70	4.15	4.60
15%	3.20	3.63	4.05	4.48	4.90
20%	3.60	4.00	4.40	4.80	5.20

Source: example proposed by the author.

We can see that the Net TS according to Miller will increase as Tps increases which is counter evident if we accept that personal taxes destroy TS. Also, the greater Tc , the greater Net TS, as expected. The behavior of Net TS with the proposed approach might be seen in Table 2b.

Table 2b. Size of Net TS, Eq (6a) or proposed approach for different Tc and Tps .

$Tc \backslash Tps$	20%	25%	30%	35%	40%
0%	2.00	2.50	3.00	3.50	4.00
5%	1.60	2.13	2.65	3.18	3.70
10%	1.20	1.75	2.30	2.85	3.40
15%	0.80	1.38	1.95	2.53	3.10
20%	0.40	1.00	1.60	2.20	2.80

Source: example proposed by the author.

These results make sense, the greater Tps the lower Net TS. The higher Tc , the higher Net TS.

In Example 2 we examine the behavior of Net TS under the two approaches: one, Miller's approach as in eq. (8b) and the other is the proposed approach as in eq. (6a), under changing X , or what is the same, under changing dividends.

Example 2

Assume $EBIT = X = 20$; $Tc = 30\%$; $D = 100$; $Kd = r = 10\%$ and $Tps = 10\%$

From that information we can derive different cash flows, as follows

Table 3. Cash flows and total cash flows: No personal taxes

FCF = EBIT(1-Tc) = X(1-Tc)	14.000
TS = rDT	3.000
FCF + TS = X(1-Tc) + KdDT	17.000
CFE = Div = (EBIT-KdD)(1-Tc) = (X-KdD) (1-Tc)	7.000
Int = CFD = KdD	10.000
CFE + CFD = (X-KdD)(1-Tc) + KdD	17.000
Total cash flows = X(1-Tc) + KdDT = (X-KdD)(1-Tc) + KdD	17.000

Source: example proposed by the author.

Table 4. Net TS according to proposed procedure and to Miller

Proposed approach, eq. (6a)	
CFETps = DivTps = (X-KdD)(1-Tc)Tps	0.700
Net TS = KdDTc-(X-KdD)(1-Tc)Tps = TS - TpsDiv	2.300
Miller's approach eq. (8b)	
KdDTc(1-Tps)+TpsKdD	3.700
Difference	
Net TS(Miller) - Net TS	1.400
X(1-Tc)Tps	1.400

Source: example proposed by the author.

Now we can examine what happens with our proposal, eq. (6a) and Miller's proposal (eq 8b) when changing X (EBIT), given the variables defined above in this example 2.

Table 5. Comparison between Miller's (Eq (8b)) and present proposal (Eq (6a))

EBIT (X)	Eq (8b)	Eq (6a)	TS	DivTps	Net TS = TS - TpsDiv	X(1-Tc)Tps	Eq (8b) - Eq (6a)
10.00	3.700	3.000	3.000	0.000	3.000	0.700	0.700
20.00	3.700	2.300	3.000	0.700	2.300	1.400	1.400
40.00	3.700	0.900	3.000	2.100	0.900	2.800	2.800
80.00	3.700	-1.900	3.000	4.900	-1.900	5.600	5.600
100.00	3.700	-3.300	3.000	6.300	-3.300	7.000	7.000
120.00	3.700	-4.700	3.000	7.700	-4.700	8.400	8.400

Source: example proposed by the author.

In Table 5 we observe that Net TS with Miller's proposal (Eq. (8b)) is constant for changes in X! Meanwhile, our proposal (Eq. (6a)) reflects the full effect of personal taxes on dividends. The higher X, the higher the dividends and the higher personal tax that will reduce TS, as expected. Also, we can observe that the difference between Eq (8b) and Eq (6a) is exactly X(1-Tc)Tps. Where this difference goes in Miller's model? As it involves X(1-Tc), it is assumed to reduce FCF and not TS. When calculating values, what is reduced is V^{Un} (the

unlevered firm value) and not Net VTS, the value of Net TS. In Miller's approach appear an after personal taxes V^{Un} . There is no such after personal tax unlevered value. Personal taxes do not tax Net Operating Income or EBIT. Remember that this *is part of personal taxes on dividends* and should affect TS and not Free Cash Flow (or $EBIT(1-T_c)$).

In the following example we examine both approaches with changing debt.

Example 3

Assume $EBIT = X = 20$, $T_c = 30\%$, $D = 100$, $K_d = r = 10\%$ and $T_{ps} = 10\%$ (**Base case**).

Now we analyze what happens with Net TS when debt D , changes and Net Ts is eq (6a) or proposed approach.

Table 6a. Net TS under proposed approach (Eq. (6a))					
Tps D	0%	5%	10%	20%	30%
0.00	0.000	-0.70	-1.400	-2.800	-4.200
20.00	0.600	-0.030	-0.660	-1.9200	-3.180
40.00	1.200	0.640	0.080	-1.0400	-2.160
50.00	1.500	0.975	0.450	-0.600	-1.650
90.00	2.700	2.315	1.930	1.1600	0.390
100.00	3.000	2.650	2.300	1.600	0.900
150.00	4.500	4.325	4.150	3.800	3.450
200.00	6.000	6.000	6.000	6.000	6.000

Source: example proposed by the author.

If we analyze what happens with Net TS when debt D changes and Net Ts is eq (8b) or Miller's approach, we find the following at Table 6b.

Table 6b. Net TS under Miller's approach (Eq. (8b))					
Tps D	0%	5%	10%	20%	30%
0.00	0.000	0.000	0.000	0.000	0.000
20.00	0.600	0.670	0.740	0.880	1.020
40.00	1.200	1.340	1.480	1.760	2.040
50.00	1.500	1.675	1.850	2.200	2.550
90.00	2.700	3.015	3.330	3.960	4.590
100.00	3.000	3.350	3.700	4.400	5.100
150.00	4.500	5.025	5.550	6.600	7.650
200.00	6.00	6.700	7.400	8.800	10.200

Source: example proposed by the author.

For Miller's proposal, Net TS *increases*, this is, the greater Tps, the greater Net TS. This is, as said, counter evident. Obviously, the greater, D, the greater Net TS. Is this logical? Yes and makes sense.

What they do is to group in the total cash flow equation as

Total Cash Flows (RHS of total cash flows equation after personal taxes)

$$(X - K_d D)(1 - T_c)(1 - T_{ps}) + K_d D(1 - T_{ps}) \quad (9)$$

From this equation they only keep the following as related to TS:

$$K_d D(1 - T_{ps}) - K_d D(1 - T_c)(1 - T_{ps}) \quad (10)$$

And $-X(1 - T_c)T_{ps}$ is considered as personal taxes on EBIT or Net Operating Income (NOI) as defined, for example by Copeland and Weston (1992, p. 452). There is no such thing as after personal tax EBIT. The missing element in (1) and (8b), $-X(1 - T_c)T_{ps}$ belongs to personal taxes on dividends, it is a reduction of Cash Flow to Equity that is mistakenly associated to EBIT and not to personal taxes. Incidentally, this approach makes equation (1) more elegant.

We can arrive to the same equation with the approach suggested in this work. The only difference is that we consider the "after personal taxes EBIT" not as part of Free Cash Flow (EBIT or NOI) but as part of personal taxes as it should be and that affects the TS.

Now we compare the original Miller proposal with our proposal to reconcile the differences, as follows:

Example 4

Assume EBIT = X = 20, T_c = 30%, D = 100, K_d = r = 10% and T_{ps} = 10% and D = 100. (**Base case**).

Table 7. Conciliation between Miller's proposal and present proposal (including T_{pb})

X	Miller Net TS	IVP Net TS = TS - DivTps - rDTpb	Miller -IVP	X(1-T _c)T _{ps}	IVP Net TS = TS - DivTps - rDTpb + X(1-T _c)T _{ps}	Diff
10	2,70	2,00	0,70	0,70	2.7	0.0
20	2,70	1,30	1,40	1,40	2.7	0.0
40	2,70	-0,10	2,80	2,80	2.7	0.0
80	2,70	-2,90	5,60	5,60	2.7	0.0
100	2,70	-4,30	7,00	7,00	2.7	0.0
120	2,70	-5,70	8,40	8,40	2.7	0.0

As can be seen in our previous four examples, there is an underestimation of personal tax effect on TS when Miller's approach is applied. Observe how our proposal for Net TS (eq (6a), Table 5)) decreases with an increase in X (EBIT) and how Miller's proposal results in a constant Net TS, (eq. (8b), Table 5), which is counter evident. Also, in tables 6a and 6b, we examine Net TS with changing debt. Under Miller's approach (Table 6b) is systematically greater than Net TS under proposed approach (Table 6a). We can also observe that in Table 6b, Miller's Net TS increases when increasing Tps. Again, this is not consistent with the idea of personal taxes affecting TS. On the other hand, if we observe Table 6a, we can propose a rule for deciding what to do when Net TS is negative. It is clear that the rule is to try to increase Net TS with more debt. In table 7, we reconcile Miller's and present approach (including Tpb) and we find that the difference is always $X(1-T_c)Tps$, that is missing in Miller's approach and attributed to the unlevered cash flow (FCF or $X(1-T_c)$).

Section Five. Value of Tax Shields (VTS) After Personal Taxes

To examine this issue from the point of view of value, assume that TS are discounted at ψ , the discount rate for TS.

Before personal taxes, VTS is

$$VTS = KdDTc/\psi \quad (11a)$$

After personal taxes on dividends Net VTS is

$$Net\ VTS = TcKdD(1 - Tps)/\psi + KdDTps/\psi - X(1-Tc)Tps/\psi \quad (11b)$$

If we assume $\psi = Ku$ we have

$$VTS = KdDTc/Ku \quad (12a)$$

After personal taxes on dividends Net VTS and using (8b) is

$$Net\ VTS = TcKdD(1 - Tps)/Ku + KdDTps/Ku - X(1-Tc)Tps/Ku \quad (12b)$$

Simplifying and recalling that $X(1-T_c)$ when discounted at Ku is V^{Un} , hence,

$$Net\ VTS = TcKdD(1 - Tps)/Ku + KdDTps/Ku - V^{Un}Tps \quad (12c)$$

Miller and his followers making simplifications and grouping terms get, as said, the elegant equation as in equation (3a), the Net VTS is

$$G = \left[1 - \frac{(1-T_c)(1-T_{ps})}{(1-T_{pb})} \right] \frac{KdD(1-T_{pb})}{r} \quad (3a)$$

This Gain or Net VTS a la Miller, assumes that market adjusts the book value of debt with personal taxes on interest received by the debt holder. As said before, market value of debt depends on the market rate of cost of debt and not on personal taxes on interest paid.

Those who accept Miller's proposal relate the last term – $V^{Un}Tps$ to V^{Un} and not to Net VTS. There is no reason to associate to V^{Un} something that by definition should be associated to Net TS and Net VTS. The effect is that what is reduced is the free cash flow, FCF, and V^{Un} and not Net VTS. Hence, Net TS and Net VTS are over estimated and it is this what causes the inconsistent behavior of Net TS (and Net VTS) as we have shown above (if we assume Ku as the discount rate of TS).

If the discount rate for TS were not Ku , but say, r , part of the Net TS is discounted at r and the other $(-X(1-Tc)Tps)$ at Ku which creates an additional inconsistency to Miller's proposal. As said, all this explains the counter evident behavior of Net TS in Miller's proposal: it misses to consider the full effect of personal taxes on dividends, this is, $X(1-Tc)Tps$ on TS. Observe that Net TS and Net VTS with Miller's proposal are overestimated and because they assume Net VTS is independent from X and the greater Tps , the greater Net TS and Net VTS which is clearly inconsistent.

This, (12c) compares with Miller (1977) proposal which discounts the Net TS at the cost of debt r :

$$\text{Net VTS (Miller)} = \left[1 - \frac{(1-T_c)(1-T_{ps})}{(1-T_{pb})} \right] \frac{KdD(1-T_{pb})}{r} \quad (3a)$$

If we redefine Miller proposal disregarding T_{pb} , we have

$$\text{Net VTS} = \frac{DKd}{r} [1 - (1 - T_c)(1 - T_{ps})] = \frac{DKd}{r} [T_c(1 - T_{ps}) + T_{ps}] \quad (13)$$

If we examine the firm value under the assumption of Ku as a discount rate of AI, we find the following:

1. According to our proposal, the firm value after the effect of personal taxes is defined in (12c).

Firm Value =

$$V^{Un} + \text{Net TS (IVP)} = V^{Un} + TcKdD(1 - Tps)/Ku + KdDTps/Ku - V^{Un}Tps$$

$$= V^{Un}(1 - Tps) + TcKdD(1 - Tps)/Ku + KdDTps/Ku \quad (14)$$

Observe that the idea of an unlevered value after personal taxes comes from a mathematical manipulation of the different elements of firm value, including the Net TS. It is not that Free Cash Flow is taxed with personal taxes. It does not make any sense. It is that $V^{Un}Tps$ is part of personal taxes on dividends as we have said repeatedly.

2. According to Miller's proposal (without Tpb) assuming that **the owner of the debt and the shareholder are different people and assuming that the net TS**, is (8b), the value of the firm is, assuming r as the discount rate of TS,

Firm Value =

$$V^{Un} + \text{Net VTS (Miller)} = V^{Un} + [Tc(1 - Tps) + Tps] KdD/r \quad (15)$$

However, if the missing part of the personal taxes on dividends ($-X(1 - Tc)Tps$) were taken into account, $X(1 - Tc)Tps$, the firm value according to Miller should be

Firm Value =

$$\begin{aligned} V^{Un} + \text{Net VTS (Miller)} &= V^{Un} + D[Tc(1 - Tps) + Tps] - X(1 - Tc)Tps/r \\ &= V^{Un} + D[Tc(1 - Tps) + Tps] - V^{Un}KuTps/r \\ &= V^{Un}(1 - KuTps/r) + D[Tc(1 - Tps) + Tps] \end{aligned} \quad (16)$$

Observe again, that the idea of an unlevered value after personal taxes comes from a mathematical manipulation of the different elements of firm value, including the Net TS.

In this analysis we have assumed that TS and Net TS are discounted at the same discount rate, either r or Ku . We also consider the TS and Net TS as a whole entity with personal taxes completely included.

We present five main findings: First, Miller's proposal (3b) assumes that Net TS and Net VTS are constant given D , Tc , Tps and Tpb . This is, the effect of personal tax, with Tps on dividends (cash flow and value) does not depend on X , or what is the same, is independent from dividends, which is counter evident, (see Table 5). Second, under the proposed approach, the greater X , the greater dividends and the greater tax on dividends and the lesser the value of net tax shields from interest (see Table 5). This is evident and expected. Third, we have observed that Miller's proposal for Net TS increases with Tps , which is counter

evident (see Tables 2a and 6b). Fourth, our proposal for Net TS and its value, decreases with increasing Tps as expected (see Tables 2b and 6a). Fifth, Miller's proposal systematically overvalues Net TS and Net VTS after personal taxes (see Tables 6a and 6b). In Table 7 we reconcile the Net Tax Shields from our proposal and Miller's. We have illustrated what is said about equations (8b) and (6a) with four examples above.

Section Six. Simplified Empirical Analysis with Companies Traded in Colombian Stock Exchange

We define as individual investor a person who obtains most of its income from capital, which has invested in various fields representing a profit, usefulness, performance or remuneration. Among the individual investors there are people who lend money to individuals or who have savings accounts, CDs, stocks, bonds, and other securities of investment. Assuming this definition and that on the average all investors have the same tax treatment, we found for 2012 that the average tax rate of individual investor, Tps is 15.87% as shown in table 8. The same for firms: the average corporate tax rate, Tc, is 32.27%, as shown in table 8.

In Colombia dividends are not taxed, in general. The next analysis is done *as if* in Colombia dividends were taxed on a personal basis.

Table 8. Tax rate for individual investors and firms, 2012

Tax rate 2012	
Average individual investor, Tps	15.87%
Average firms, Tc	32.27%
Tc/Tps	2.03

*Source: http://www.dian.gov.co/dian/14cifrasgestion.nsf/pages/Aggregados_declaraciones_tributarias?OpenDocument and author's calculations.

If we assume an average individual investor and an average firm, they will have an average tax rate as shown above in Table 8. And in that table we also estimate the average Tc/Tps. In this case, it is 2.03 (32.27%/15.87%).

We examined 21 firms listed in the Colombian stock market between 2000 and 2013. We selected cases (208) with interest payments. This is, those that supposedly have tax shields and had information from 2000 to 2013 and we calculated the Dividends/Interest ratio.

The firms selected for this analysis are shown in table 9.

Table 9. Traded firms in Colombian Stock Exchange included in the sample			
Fabricato	Ecopetrol	ESTRA	Cartón Colombia
Promigas	ETB	ISA	Celsia
Productos Familia	Enka	ISAGEN	Cementos Argos
ODINSA	Gas Natural	Manufacturas de cemento	Coltejer S. A.
Mineros	Nutresa	Tablemac	Corferias
Éxito			

Source: Superintendencia Financiera and Bolsa de Valores de Colombia.

We are interested on the case where personal taxes decrease TS:

$$TcInt - TpsDiv \leq 0 \quad (17a)$$

Equation (6a) is the case when TS are reduced by personal taxes.

$$TcInt \leq TpsDiv \quad (17b)$$

$$Tc/Tps \leq Div/Int \quad (17c)$$

In these cases, if we define a given Tc/Tps for an ideal, hypothetical average investor investing in an average firm, as in Table 7 and identify that level in a table with Div/Int , as in Table 10 below, for all firms with values of Div/Int above that critical value there will be destruction of TS because $Tc/Tps \leq Div/Int$. The group of firms that have Div/Int below that level will not fully destroy TS. This will give us an insight on the validity of Miller's (1977) proposal.

In next table we show the distribution of Div/Int , for 208 observations between 200 and 2013.

Table 10. Dividends/Interest for 21 firms 2000-2013			
Div/Int	Frequency	Cumulated absolute frequency	Cumulated %
1	94	94	45.19%
2	25	119	57.21%
...and more	89	208	100.00%

Source: Superintendencia Financiera and Bolsa de Valores de Colombia and author's calculations.

The previous table tells us that if the average investor in an average firm has $Tc/Tps = 2.03$, we can see that a little more than 57% of the observations (firm-year) did not destroy tax shields. This means that the generalizations made when using Miller's approach would not be valid for more than 57% of the cases.

Pension funds are large investors in traded and non-traded companies and are tax-free entities in Colombia, as in many other countries. They are not included, nor identified in the aggregates. Here we assume that all companies pay taxes. If pension funds data are taken into account, the number of firms where personal taxes paid by shareholders do not fully destroy their TS will increase.

Section 7. Summary and Conclusion

We have discussed the relevance of personal taxes on interest received by debt holders, Tpb and the proper way to determine the Net TS and Net VTS after personal taxes on dividends. We have illustrated that Miller's approach underestimates the effect of personal taxes on debt tax shields, TS and have some inconsistencies; in the same vein, that Miller's approach relates part of the personal taxes to FCF and not to Net TS.

We conclude that personal taxes on dividends should be considered as an independent and complete entity without subtracting part of those personal taxes to an item different to TS and subtract personal taxes on dividends from TS to define Net TS after personal tax on dividends.

Future work is needed to test these ideas and define how much TS is destroyed by personal taxes on dividends and if Net TS and Net VTS are positive, zero or negative that is equivalent to calculate the loss in TS due to personal taxes. We have suggested an approach to test Miller's proposal and its followers. For measuring the loss of TS we recommend calculating directly it as Dividends (an information available in public financial statements) paid times Tps and discounting that at the proper discount rate to find its value as part of firm value. We elaborated an example on how could the information be included in the analysis.

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