# Simple Errors or Myths in Dealing with Perpetuities

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## Abstract

Terminal value is critical for valuation purposes because very often it is a large part of what constitutes the value of a firm.

In this short note I answer and clarify some typical questions and myths related to the calculation of terminal value. They are related to the use of non growing perpetuities, inflation and real growth; the use of Net Operating Profits Less Adjusted Taxes, NOPLAT as a proxy to the Free Cash Flow in perpetuity; the use of the typical textbook formula for estimating terminal value; and the treatment of working capital in perpetuities.

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# Simple Errors or Myths in Dealing with Perpetuities

> "I have a doubt: if we have neither deflated the forecasted cash flows, nor deflated the WACC in order to bring them back to present value, why deflate WACC for perpetuity and not deflate the cash flow for the same perpetuity? It seems to me that there is some inconsistency here". (Anonymous), June, 2009

## Introduction

In several previous papers I have dealt with the issue of perpetuities, its present value and the discount rate. My conclusion has been that perpetuities are a Pandora's Box. After the discussion of the issue I think that the best solution for a consistent terminal value is to assume a simple non growing perpetuity.

In this teaching note I discuss some typical questions posed by students and teachers regarding perpetuities. My concern is with some recurrent questions:

1. When using non growing perpetuities why use a constant non deflated Free Cash Flow, FCF and a real, inflation free discount rate?
2. What is wrong with using the last cash flow, $FCF_N$ as a starting point to calculate terminal value? Which is the proper FCF to be used in calculating terminal value? Is it true we should use Net Operating Profits Less Adjusted Taxes, NOPLAT? Why if that is not a cash flow?
3. What is wrong with the popular formula for terminal value $FCF_N \times (1+G)/(W-G)$?
4. TV is calculated as a growing perpetuity, thus the working capital will be also growing from the number at the end of the planning horizon. Why do you suggest recovering it at the end of the planning horizon?

The purpose of this note is to shed light upon these questions.

## Questions and Comments

### Question One

When using non growing perpetuities why use a constant non deflated FCF and a real, inflation free discount rate and not a nominal discount rate?

When calculating terminal value assuming simple non growing perpetuity means that the FCF will not grow at all. This is equivalent to say that nominal growth, G is zero. With exception of a pathological and strange situation in which real growth, g and inflation has different signs and values to get G=0, usually that means that real growth, g and inflation rate are zero. Assume that $FCF_N$ is 1,000 and we say that it is a non growing perpetuity. Which will be the value of FCF at N+1, N+2, etcetera? Simple: 1,000. Hence, using the $FCF_N$ from N+1 up to infinity implies no real growth and no inflation. No need to deflate that cash flow; the cash flow is in dollars of year N from N+1 to infinity. When the terminal value is discounted back to period 0, it is deflated with the inflation that is implicit in the discount rates from 1 to N.

On the other hand, if inflation is zero from N+1 to infinity, discount rates in that time interval will not have inflation in it. Remember that nominal discount rates are interest rates that have three components: inflation, real interest rate and eventually a risk premium. Assuming no risk (just to simplify the situation), the discount rate in a non inflationary environment will be the real interest rate. If the nominal discount rate has the risky component, then the discount rate would be estimated deflating the nominal discount rate.

A complementary question is why we do not deflate the discount rates during the planning horizon and we deflate only for perpetuities? During the planning horizon we usually have inflation; hence, the discount rate will have inflation and should not be deflated. Why if from N+1 to infinity why we do not deflate the discount rate? Inflation in interest rates are not cumulative, they are included in each period if inflation exists. For

instance, if inflation rate in period N is 5%, that inflation will affect the discount rate of N; if in N-1 inflation rate is 3%, that inflation will affect the discount rate for N-1. If we assume that for terminal value calculation we will not have inflation (from N+1 to infinity) hence, 0% will be the inflation rate that will affect each discount rate in that interval. Hence, the discount rate from N+1 to infinity has to be the real or deflated one.

In summary, if between 0 and N there is non negative inflation, hence the cash flows and discount rates will have an inflationary component affecting their values and we should not deflate. However, the deflation occurs when we discount the cash flows with a nominal discount rate. When there is no inflation the discount rate does not have an inflationary component and the FCF does not increase due to inflation. If in addition there is no real growth, hence the FCF will growth neither by inflation, nor by real growth and hence, the FCF will have the value it had in the previous period, this is, $FCF_N$.

**Question Two**

What is wrong with using the last cash flow, $FCF_N$ as a starting point to calculate terminal value? Which is the proper FCF to be used in calculating terminal value? Is it true we should use Net Operating Profits Less Adjusted Taxes, NOPLAT? Why if it is not a cash flow?

Free Cash Flow from 0 to N could have or not a perpetual or permanent item that accounts for replenishing the value of assets. Let us assume the simplest case for perpetuities: non growing perpetuities. If the $FCF_N$ has an investment to replenish the level of assets we could assume that the firm has a constant level of assets and we might assume that FCF will be constant. If it is not the case, we have to provide inside the FCF an item to replenish the level of assets in order to have a constant FCF.

One way to assume that assets are constant is to consider that the amount of depreciation charges is invested in assets. Although this might raise several questions such as what if we do not use a lineal depreciation method but a accelerated one and we are in the lower side of the sequence, it does not matter. Once we assume that depreciation will be reinvested in assets we are maintaining the assets level and we could assume a constant FCF. This is the crux of the assumption. In practice this is what we do if we make the value of the FCF for perpetuity equal to Net Operating Profits Less Adjusted Taxes, NOPLAT. As NOPLAT has depreciation charges subtracting, when we say that FCF is equal to NOPLAT we are assuming, among other assumptions that depreciation is invested in assets and hence the FCF will be kept constant. Notice we are not saying NOPLAT is a cash flow; what we say is that the FCF will have the value of NOPLAT, which is conceptually different.

**Question Three**

What is wrong with the popular formula for terminal value $FCF_N \times (1+G)/(W-G)$[1]?

Unless you have provided for a perpetual investment to grant growth in the $FCF_N$, G cannot be sustained from the thin air. Let us assume that $FCF_N$ is equal to $NOPLAT_N$ that grants the perpetual replenishment of assets and a constant FCF. Where growth comes from? Will it be possible for a firm to positively grow at perpetuity with the same level of assets? No, and this is an incontrovertible answer. Hence, we need to put aside part of the FCF in order to provide a perpetual growth. Intuitively, the higher the desired growth, the higher the fraction of FCF that has to be invested in assets. This is measured by the plowback ratio. This is well discussed by Miller and Modigliani (1961). The plowback ratio

[1] G is the nominal growth rate and W is the Weighted Average Cost of Capital, WACC.

will be a function of g (real growth) and w (real or deflated WACC). Hence, the popular terminal value formulation should be modified to

$$TV_N = \frac{NOPLAT_N \times (1+G) \times \left(1 - \frac{g}{w}\right)}{W-G} \quad (1)$$

Where g is real growth and w is real or deflated WACC.

Using the Fisher relation and simplifying, we arrive to

$$TV_N = \frac{NOPLAT_N \times (1+g)}{w} \quad (2)$$

If $FCF_N$ has the proper investment in assets not only to keep assets constant, but to increase them in order to grant a sustained growth, we can use the popular textbook formulation for TV which is

$$TV_N = \frac{FCF_N \times (1+G)}{W-G} \quad (3)$$

My caveats regarding the use of perpetuities are presented in several working papers. (See Vélez-Pareja, 2004, 2006a, 2006b, 2007, 2008 and Vélez-Pareja and Tham, 2007). At this time I think the only consistent approach to perpetuities is the simple, non growing perpetuity.

**Question Four**

Terminal value is calculated as a growing perpetuity, thus the working capital will be also growing from the number at the end of the planning horizon. Why do you suggest recovering it?

If we use NOPLAT as the FCF at N we have the following situation:

1. No Accounts Receivable, AR. In NOPLAT you are assuming sales revenues are received when you invoice them. The same with Accounts Payable, AP. The inventory line is implicit in the Cost of goods sold. I short, when assuming NOPLAT as FCF the working capital is implicit in the FCF: No AR, no AP and the inventory will increase with the growth for the FCF.

2. If you do not "recover" the working capital in the last period you will have that amount sunk in there AND at the same time you will include a working capital implicit in the CF because you have no AR and no AP as said above.
3. You are investing depreciation every year. Depreciation charges are subtracted in NOPLAT. Investing depreciation will keep FCF constant as discussed above.

This argument has implicit a central idea: in general, do not use FCF to calculate TV, use NOPLAT.

**Concluding Remarks**

We have explained and commented some recurrent questions and myths in dealing with perpetuities. They are more frequent than imagined and you find them in textbooks or when you exchange of information with teachers and students. Care has to be taken in dealing with perpetuities because terminal value might have a large proportion of total value.

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