Cost of Capital When Dividends are Deductible

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Abstract

When calculating Tax Savings, TS we are confronted with a strange mix of accounting accrual and market value when involving TS in the calculation of the Weighted Average Cost of Capital, WACC or the Cost of Equity, Ke. Firms earn the right to TS once they accrue the interest expense and they actually earn the TS when taxes are paid.

Tax savings and the discount rate (ψ) we use to calculate their value are involved in the calculation of WACC and Ke. Textbook WACC formulation is a very special and unique case that is not typical. Based on previous findings, we derive a general approach to those formulas that take into account any kind of TS related to the financing decision of a firm and any date when the TS is earned. These formulations can be used to introduce any type of externality that creates value through tax savings not captured by neither the cost of debt nor the cost of equity.

In this paper we develop the formulations for Ke, the cost of levered equity and the average cost of capital when dividends or interest on dividends are deductible.

We show that using the proper formulation the most known valuation methods, i) Firm value with Free Cash Flow and WACC for the FCF; ii) value with the Capital Cash Flow and WACC for the CCF; iii) equity value with the Cash Flow to Equity and Ke, the levered cost of equity plus debt; iv) Adjusted Present Value, APV are consistent and give identical results.

JEL codes: D61, G31, H43

Key words or phrases: WACC, interest on equity, tax savings, tax shields, cost of equity, deductible dividends, deductible interest on equity, discount rate for tax savings

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Introduction

When calculating Tax Savings, TS we are confronted with a strange mix of accounting accrual and market value when involving TS in the calculation of the Weighted Average Cost of Capital, WACC or the Cost of Equity, Ke. Firms earn the right to TS once they accrue the interest expense and they actually earn the TS when taxes are paid. (See, Vélez-Pareja, 2008).

Tax savings and the discount rate (ψ) we use to calculate their value are involved in the calculation of WACC and Ke. Textbook WACC formulation is a very special and unique case that is not typical. Based on previous findings, we derive a general approach to those formulas that take into account any kind of TS related to the financing decision of a firm and any date when the TS is earned. These formulations can be used to introduce any type of externality that creates value through tax savings not captured by neither the cost of debt nor the cost of equity.

Taggart (1991) developed some expressions with these purposes. He considers corporate and personal taxes; we only consider corporate taxes. There is no derivation of the formulas in his work. Tham, J. and Vélez–Pareja I. (2002) and Tham, J. and Vélez–Pareja I. (2004) derive the proper formulations for Ke and WACC. Vélez–Pareja, I., & Tham, J. (2003) use these derivations to incorporate the effect of losses in exchange rate, of losses carried forward, of unpaid taxes, of Presumptive Income Taxation and the effect of inflation adjustment of book value of equity when adjusting financial statements by inflation in tax savings. We derive the explicit and general formulation to include other sources of TS and their discount rate, ψ.

These refinements for calculating Ke and WACC are based on Modigliani & Miller propositions and they are just a tuning up of them to include idiosyncratic conditions found in different markets. In this work we study the impact of any new source of tax savings. In particular, we study the specific case of Brazil, one of the major economies in the world. Although it is an interesting case, the basic ideas posed by M&M are the same.

When Brazil used to adjust the financial statements by inflation, they (as many other economies) allowed for adjustment of book value of equity using an index linked to the inflation rate. According to Zani & Ness (2001) after many years of inflation adjustment with a charge equal to the adjustment of book value of equity, since January 1, 1996 firms were allowed to charge interest on the book value of equity and had not only the effect to be a deductible charge, but to pay those interest expenses as part of the dividends defined by the firm. What initially was an accounting accrual figure now is an effective payment to shareholders with the associated tax savings benefits as before.

In the financial report of a Brazilian firm, Aracruz Celulosa to the U.S. Securities and Exchange Commission, they say:

> "As of January 1, 1996, Brazilian corporations are allowed to attribute interest on stockholders' equity. The calculation is based on the stockholders' equity amounts as stated in the statutory accounting records and the interest rate applied may not exceed the long-term interest rate ("TJLP") determined by the Brazilian Central Bank (approximately 9.75%, 7.78% and 6.32% for years 2005, 2006 and 2007, respectively). Also, such interest may not exceed the greater of 50% of net income for the year or 50% of retained earnings plus income reserves (including those mentioned above), determined in each case on the basis of the statutory financial statements. The amount of interest attributed to stockholders is deductible for corporate income tax purposes." http://www.aracruz.com/minisites/ra2005/localaracruz/ra2005/en/if/demonstracoes_notas.html visited on June 14, 2009)

Non-traded stock corporations may pay interest on equity JSCP by its initials in Portuguese. The long term interest rate ("A Taxa de Juros de Longo Prazo - TJLP" in Portuguese) is not a market rate. It is established by the National Monetary Council (Conselho Monetário Nacional) and used for loans by the BNDES. "The Brazilian Development Bank (BNDES) is a federal public company, linked to the Ministry of Development, Industry and Foreign Trade (MDIC). Its goal is to provide long-term financing aimed at enhancing Brazil's development, and, therefore, improving the competitiveness of the Brazilian economy and the standard of living of the Brazilian population." (http://www.bndes.gov.br/english/thecompany.asp visited June 15, 2009).

This practice, apart from the adjustment for inflation that was made on an accrual basis, is unusual in the sense of becoming an actual payment, a cash flow. This is not a new cash flow, but it is part of the dividends defined by the firm and yet, they are deductible and hence, the firm earns TS on that.

The Income Statement

An Income Statement according to this Brazilian regulation would be as follows:

Earnings Before Interest and Taxes	EBIT
-Interest on Debt	$-k_d$*D
-Interest on book value of equity	$-k_f$'*E
=Earnings Before Tax	=EBT
- Income Taxes	-T*EBT
= Net Income	=NI
-Dividends	-Dividends paid
=To Retained Earnings	=Add to Retained Earnings

The financing cash flow can de disaggregated as follows:

- Cash Flow to Equity, CFE

$$CFE = EBIT(1-T)-\Delta WC -kd\times D\times (1-T)+\Delta D+k_f'\times E\times T = FCF - CFD + TS^D + TS^E \qquad (1)$$

- Cash Flow to Debt, CFD

$$k_d\times D-\Delta D = CFD \qquad (2)$$

- Capital Cash Flow, CCF

$$FCF + TS^D + TS^E = FCF + kd\times D\times T + k_f'\times BVE\times T \qquad (3)$$

It is clear that the CFE is increased by TS_{equity} and this fact has to be included in the derivation of Ke and WACC.

Variables in equations

$WACC_{gen}$ = Weighted Average Cost of Capital in a general formulation.
Rm = Market return
Rf = Risk free rate
MRP = Market Risk Premium = Rm - Rf
Ku = Cost of unlevered equity and can be calculated using CAPM or any other procedure, Ku = Rf $+\beta_u\times$(Rm-Rf)
Ke= Cost of levered equity
ψ^E = Discount rate for tax savings from equity interest.
ψ^D = Discount rate for tax savings from debt interest
Kd = Cost of debt
T= Corporate Tax Rate
FCF = Free Cash Flow

General Formulations for Ke, WACC for the FCF and for the CCF

In this Section we develop the formulations for the cost of capital taking into account the tax savings when interest on equity (or dividends) are deductible.

The Cost of Levered Equity

The general expression for Ke is

$$Ke_t = Ku_t + \left(Ku_t - Kd_t\right)\frac{D_{t-1}}{E_{t-1}} - \left(Ku_t - \psi^D_t\right)\frac{V^{TSD}_{t-1}}{E_{t-1}} - \left(Ku_t - \psi^E_t\right)\frac{V^{TSE}_{t.1}}{E_{t-1}} \quad (4)$$

If $\psi^D = \psi^E = Ku$ then

$$Ke_t = Ku_t + \left(Ku_t - Kd_t\right)\frac{D_{t-1}}{E_{t-1}} \quad (5a)$$

If $\psi^D = \psi^E = Kd$ then

$$Ke_t = Ku_t + \left(Ku_t - Kd_t\right)\left[\frac{D_{t-1}}{E_{t-1}} - \frac{V^{TSD}_{t-1}}{E_{t-1}} - \frac{V^{TSE}_{t-1}}{E_{t-1}}\right] \quad (5b)$$

If $\psi^D = Kd$ and $\psi^E = Ke$ then

$$Ke_t = Ku_t + \left(Ku_t - Kd_t\right)\left[\frac{D_{t-1} - V^{TSD}_{t-1}}{E_{t-1} - V^{TSE}_{t-1}}\right] \quad (5c)$$

Observe that we can obtain a constant Ke only when leverage is constant and $\psi^D = \psi^E = Ku$.

In summary

Formula	CFE
General	$Ke_t = Ku_t + (Ku_t - Kd_t)\frac{D_{t-1}}{E_{t-1}} - \left(Ku_t - \psi^D_t\right)\frac{V^{TSD}_{t-1}}{E_{t-1}} - \left(Ku_t - \psi^E_t\right)\frac{V^{TSE}_{t.1}}{E_{t-1}}$
$\psi^D = \psi^E = Ku$	$Ke_t = Ku_t + (Ku_t - Kd_t)\frac{D_{t-1}}{E_{t-1}}$
$\psi^D = \psi^E = Kd$	$Ke_t = Ku_t + (Ku_t - Kd_t)\left[\frac{D_{t-1}}{E_{t-1}} - \frac{V^{TSD}_{t-1}}{E_{t-1}} - \frac{V^{TSE}_{t-1}}{E_{t-1}}\right]$
$\psi^D = Kd$ and $\psi^E = Ke$	$Ke_t = Ku_t + (Ku_t - Kd_t)\left[\frac{D_{t-1} - V^{TSD}_{t-1}}{E_{t-1} - V^{TSE}_{t-1}}\right]$

The General Formulation for Weighted Average Cost of Capital for the FCF

The general expression for $WACC_{gen}$ for the FCF is

$$WACC^{FCF}_{gen\ t} = Ku_t - \left(Ku_t - \psi^D_t\right)\times\frac{V^{TSD}_{t-1}}{V^L_{t-1}} - \left(Ku_t - \psi^E_t\right)\times\frac{V^{TSE}_{t-1}}{V^L_{t-1}} - \frac{TS^D_t}{V^L_{t-1}} - \frac{TS^E_t}{V^L_{t-1}} \quad (6)$$

If $\psi^D = \psi^E = Ku$ then

$$WACC^{FCF}_{gen\ t} = Ku_t - \frac{TS^D_t}{V^L_{t-1}} - \frac{TS^E_t}{V^L_{t-1}} \quad (7a)$$

If $\psi^D = \psi^E = Kd$ then

$$WACC^{FCF}_{gen\ t} = Ku_t - (Ku_t - Kd_t)\times\left[\frac{V^{TSD}_{t-1}}{V^L_{t-1}} + \frac{V^{TSE}_{t-1}}{V^L_{t-1}}\right] - \frac{TS^D_t}{V^L_{t-1}} - \frac{TS^E_t}{V^L_{t-1}} \quad (7b)$$

If $\psi^D = Kd$ and $\psi^E = Ke$ then

$$WACC^{FCF}_{gen\ t} = Ku_t - (Ku_t - Kd_t)\times\frac{V^{TSD}_{t-1}}{V^L_{t-1}} - (Ku_t - Ke_t)\times\frac{V^{TSE}_{t-1}}{V^L_{t-1}} - \frac{TS^D_t - TS^E_t}{V^L_{t-1}} \quad (7c)$$

Observe that with this assumption we cannot obtain a constant WACC when leverage is constant.

In summary

Formula	FCF
General	$\text{WACC}_{\text{gen t}}^{\text{FCF}} = \text{Ku}_t - \left(\text{Ku}_t - \psi_t^D\right) \times \frac{V_{t-1}^{TSD}}{V_{t-1}^L} - \left(\text{Ku}_t - \psi_t^E\right) \times \frac{V_{t-1}^{TSE}}{V_{t-1}^L} - \frac{TS_t^D}{V_{t-1}^L} - \frac{TS_t^E}{V_{t-1}^L}$
$\psi^D = \psi^E = \text{Ku}$	$\text{WACC}_{\text{gen t}}^{\text{FCF}} = \text{Ku}_t - \frac{TS_t^D}{V_{t-1}^L} - \frac{TS_t^E}{V_{t-1}^L}$
$\psi^D = \psi^E = \text{Kd}$	$\text{WACC}_{\text{gen t}}^{\text{FCF}} = \text{Ku}_t - (\text{Ku}_t - \text{Kd}_t) \times \left[\frac{V_{t-1}^{TSD}}{V_{t-1}^L} + \frac{V_{t-1}^{TSE}}{V_{t-1}^L}\right] - \frac{TS_t^D}{V_{t-1}^L} - \frac{TS_t^E}{V_{t-1}^L}$
$\psi^D = \text{Kd}$ and $\psi^E = \text{Ke}$	$\text{WACC}_{\text{gen t}}^{\text{FCF}} = \text{Ku}_t - (\text{Ku}_t - \text{Kd}_t) \times \frac{V_{t-1}^{TSD}}{V_{t-1}^L} - (\text{Ku}_t - \text{Ke}_t) \times \frac{V_{t-1}^{TSE}}{V_{t-1}^L} - \frac{TS_t^D - TS_t^E}{V_{t-1}^L}$

The General Formulation for Weighted Average Cost of Capital for the CCF

The general expression for WACC_{gen} for the CCF is

$$\text{WACC}_{\text{gen t}}^{\text{CCF}} = \text{Ku}_t - \left(\text{Ku}_t - \psi_t^D\right) \times \frac{V_{t-1}^{TSD}}{V_{t-1}^L} - \left(\text{Ku}_t - \psi_t^E\right) \times \frac{V_{t-1}^{TSE}}{V_{t-1}^L} \tag{8}$$

- If $\psi^D = \psi^E = \text{Ku}$ then

$$\text{WACC}_{\text{gen t}}^{\text{CCF}} = \text{Ku}_t \tag{9a}$$

- If $\psi^D = \psi^E = \text{Kd}$ then

$$\text{WACC}_{\text{gen t}}^{\text{CCF}} = \text{Ku}_t - (\text{Ku}_t - \text{Kd}_t) \times \left[\frac{V_{t-1}^{TSD}}{V_{t-1}^L} + \frac{V_{t-1}^{TSE}}{V_{t-1}^L}\right] \tag{9b}$$

- If $\psi^D = \text{Kd}$ and $\psi^E = \text{Ke}$ then

$$\text{WACC}_{\text{gen t}}^{\text{CCF}} = \text{Ku}_t - (\text{Ku}_t - \text{Kd}_t) \times \frac{V_{t-1}^{TSD}}{V_{t-1}^L} - (\text{Ku}_t - \text{Ke}_t) \times \frac{V_{t-1}^{TSE}}{V_{t-1}^L} \tag{9c}$$

In summary

Formula	CCF
General	$\text{WACC}_{\text{gen t}}^{\text{CCF}} = \text{Ku}_t - \left(\text{Ku}_t - \psi_t^D\right) \times \frac{V_{t-1}^{TSD}}{V_{t-1}^L} - \left(\text{Ku}_t - \psi_t^E\right) \times \frac{V_{t-1}^{TSE}}{V_{t-1}^L}$
$\psi^D = \psi^E = \text{Ku}$	$\text{WACC}_{\text{gen t}}^{\text{CCF}} = \text{Ku}_t$
$\psi^D = \psi^E = \text{Kd}$	$\text{WACC}_{\text{gen t}}^{\text{CCF}} = \text{Ku}_t - (\text{Ku}_t - \text{Kd}_t) \times \left[\frac{V_{t-1}^{TSD}}{V_{t-1}^L} + \frac{V_{t-1}^{TSE}}{V_{t-1}^L}\right]$
$\psi^D = \text{Kd}$ and $\psi^E = \text{Ke}$	$\text{WACC}_{\text{gen t}}^{\text{CCF}} = \text{Ku}_t - (\text{Ku}_t - \text{Kd}_t) \times \frac{V_{t-1}^{TSD}}{V_{t-1}^L} - (\text{Ku}_t - \text{Ke}_t) \times \frac{V_{t-1}^{TSE}}{V_{t-1}^L}$

The derivation of these formulas can be seen in the appendix.

Observe that we can obtain a constant WACC for the CCF, $WACC^{CCF}$, only when leverage is constant and $\psi^D = \psi^E = Ku$.

A Finite Cash Flows Example

Next we show a finite cash flow example. In this example we consider four methods to calculate value: DCF with Ke for the CFE; DCF with WACC for the FCF; DCF with the WACC for the CCF; and Adjusted Present Value, APV for three scenarios for the discount rate of tax shields.

Input data 1. Basic Inputs: Beta and rates

T	40%	βu	1
Kd	12%	Rf	7%
Interest on equity'	8%	Rm-Rf	7%
		Ku	14.0%

Another input data is related to the cash flows and debt balances.

Input data 2 Cash Flows

	0	1	2	3	4	5
Debt, D	100.00	80.00	60.00	40.00	20.00	-
Debt Payment		20.00	20.00	20.00	20.00	20.00
Interest on D		12.00	9.60	7.20	4.80	2.40
CFD		32.00	29.60	27.20	24.80	22.40
TSD		4.80	3.84	2.88	1.92	0.96
FCF		40.00	42.00	44.10	46.31	48.62
BVE	100.00	100.00	100.00	100.00	100.00	100.00
Interest on BVE		8.00	8.00	8.00	8.00	8.00
TSE		3.20	3.20	3.20	3.20	3.20
CCF		48.00	49.04	50.18	51.43	52.78
CFE		16.00	19.44	22.98	26.63	30.38

With this information we calculate the firm value for different scenarios of the discount rate for TS.

Value Calculations 1. Discount Rate for TS = Ku

If $\psi^D = \psi^E = Ku$						
	0	1	2	3	4	5
Ke		16.79%	16.37%	16.03%	15.75%	15.52%
E	71.57	67.59	59.21	45.72	26.30	
V=D+E	171.57	147.59	119.21	85.72	46.30	
$WACC^{FCF}$		9.34%	9.23%	8.90%	8.03%	5.01%
V	171.57	147.59	119.21	85.72	46.30	
D	100.000	80.000	60.000	40.000	20.000	-
PV CCF	171.57	147.59	119.21	85.72	46.30	
V^{Un}	149.84	130.82	107.13	78.03	42.65	
V^{TSD}	10.74	7.45	4.65	2.42	0.84	
V^{TSE}	10.99	9.32	7.43	5.27	2.81	
APV	171.57	147.59	119.21	85.72	46.30	

Value Calculations 2. Discount Rate for TS = Kd

If $\psi^D = \psi^E = Kd$	0	1	2	3	4	5
VTSD	11.16	7.70	4.79	2.48	0.86	
VTSE	11.54	9.72	7.69	5.41	2.86	
Ke		16.13%	15.83%	15.59%	15.40%	15.24%
E	72.54	68.24	59.60	45.92	26.36	
V=D+E	172.54	148.24	119.60	85.92	46.36	
$WACC^{FCF}$		9.10%	9.02%	8.71%	7.86%	4.87%
V	172.54	148.24	119.60	85.92	46.36	
D	100.000	80.000	60.000	40.000	20.000	
$WACC^{CCF}$		13.74%	13.76%	13.79%	13.82%	13.84%
PV CCF	172.54	148.24	119.60	85.92	46.36	
V^{Un}	149.84	130.82	107.13	78.03	42.65	
V^{TSD}	11.16	7.70	4.79	2.48	0.86	
V^{TSE}	11.54	9.72	7.69	5.41	2.86	
APV	172.54	148.24	119.60	85.92	46.36	

Value Calculations 3. Discount Rate for TS = Kd and Ke

If ψ^D = Kd ψ^E = Ke	0	1	2	3	4	5
VTSD	11.16	7.70	4.79	2.48	0.86	
VTSE	10.37	8.92	7.19	5.15	2.77	
Ke		16.91%	16.47%	16.13%	15.85%	15.63%
E	71.37	67.44	59.11	45.66	26.27	
V=D+E	171.37	147.44	119.11	85.66	46.27	
$WACC^{FCF}$		9.38%	9.27%	8.94%	8.08%	5.07%
V	171.37	147.44	119.11	85.66	46.27	
D	100.000	80.000	60.000	40.000	20.000	
$WACC^{CCF}$		14.05%	14.05%	14.05%	14.05%	14.06%
PV CCF	171.37	147.44	119.11	85.66	46.27	
V^{Un}	149.84	130.82	107.13	78.03	42.65	
V^{TSD}	11.16	7.70	4.79	2.48	0.86	
V^{TSE}	10.37	8.92	7.19	5.15	2.77	
APV	171.37	147.44	119.11	85.66	46.27	

As can be seen for each set of assumptions on discount rate for TS, the four methods give consistent and identical results as expected.

Although it is not the purpose of this paper, we can say something regarding the risk for the tax savings. In case of debt, we have two situations:

1. Case 1. Leverage D% is defined as a target leverage and Debt = D%xV_{t-1} and V depends on FCF; hence the risk of TS should be Ku.
2. Case 2. Debt and the CFD profile are defined. Again, TS dpends on FCF (or EBIT). In Vélez Pareja (2009) it is shown that if EBIT > Financial Expenses (FE) TS = T×FE; if 0<EBIT<GF, TS = T×EBIT; if EBIT<0, TS = 0. Hence, TS depends on EBIT and it risk should be Ku, the cost of unlevered equity (that is the risk of FCF).

Concluding Remarks

This paper analyzed the formulation of WACC and Ke under scenarios with tax savings originated by items different than the interest charges on debt. We have derived the formulations in a general way and they can be used for finite and perpetuities cash flows. For the later, we have to recognize the value of TS in a given perpetuity scenario. We show an example for finite cash flows. In this example we show that four methods give consistent results: i) Firm value with Free Cash Flow, FCF and WACC for the FCF, $WACC^{FCF}$; ii) value with the Capital Cash Flow, CCF; iii) equity value with the Cash Flow to Equity and Ke, the levered cost of equity plus debt; iv) Adjusted Present Value, APV.

We calculated value for three scenarios depending on the discount rate ψ for TS from two sources: interest charges on debt and interest charges on book value of equity. The value for ψ was Kd and Ku for both tax savings and a third one that assumes Kd and Ke for TS from debt and equity respectively.

The formulations work for any debt profile: constant debt, variable debt or constant or target leverage. From the formulations as mentioned in Vélez-Pareja, Ibragimov and Tham, 2008, constant leverage does not grant that WACC or Ke be constant. It depends on how the TS affect the respective formulation. The only formulas whose value remains constant with constant leverage are Ke and $WACC^{CCF}$ when the discount rate ψ, for both tax savings is equal to Ku.

Finally, we analyze which should be the proper discount rate for the two tax savings (debt and book value of equity). We suggest that it should be the cost of unlevered equity, Ku.

For further research we might examine the effect of tax shields earned by equity (interest on equity or adjustment of equity when financial statements are adjusted by inflation) on the capital structure in different countries including Brazil with the two scenarios: adjustment of equity (when Brazil used to adjust financial statements by inflation) and interest on equity (as a part of dividends paid).

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Appendix

Derivation of Traditional Textbook formula for WACC for the FCF

The discount rate for free cash flows is the weighted average cost of capital WACC. We derive its formulation using a modified version of the equilibrium equation for values. Firm value is (Exhibit 1) the value of operations, Vu plus the value of TS from debt (V^{TSD}) plus the value of TS for equity (V^{TSE}). This value should be identical to the value of debt holders plus equity. Each element is associated to a discount rate according to its risk profile.

<table>
<tr><td>V^{Un}
(Ku)</td><td rowspan="2">D
(Kd)</td></tr>
<tr><td>V^{TSD}
(ψ^D)</td></tr>
<tr><td>V^{TSE}
(ψ^E)</td><td>E
(Ke)</td></tr>
</table>

Exhibit 1. The firm in terms of assets and fund providers.

The Cost of Equity, Ke

The treatment for perpetuities and finite cash flows is basically the same. When we deal with finite cash flows we relate a value at t with its value at t+1 multiplying the value at t by (1+discount rate). However, when we realize that Vu + VTSd + VTSe = D + E, the 1 inside the parenthesis multiplies exactly these values at the left and the right hand sides of the equation and they cancel each other. The net result is that the formulation is the same for finite cash flows and for perpetuities. Care has to be taken when calculating the values involved in the formulation. In particular, when we are calculating the value of the tax shields, for instance, the value of tax savings for interest expenses when we deal a perpetuity and the discount rate for the TS is Kd. The TS is T×Dt-1×Kdt. If we deal with a perpetuity, the present value of a perpetuity of T×Dt-1×Kdt is T×Dt-1×Kdt/Kdt = T×Dt-1. When dealing with finite cash flows we do not know the value in a standard from as in te perpetuity and has to be calculated depending on the future TS within the planning horizon.

$$FCF_t + TS_t^D + TS_t^E = CFD_t + CFE_t \quad (A1)$$

$$Vu_t + V_t^{TSD} + V_t^{TSE} = D_t + E_t \quad (A2)$$

$$Vu_t = D_t + E_t - V_t^{TSD} - V_t^{TSE} \quad (A3)$$

$$Vu_{t-1}\times Ku_t + V^{TSD}_{t-1}\times\psi^D + V^{TSE}_{t-1}\times\psi^E = D_{t-1}\times Kd_t + E_{t-1}\times Ke_t \qquad (A4a)$$

Solving for Ke_t

$$E_{t-1}\times Ke_t = Vu_{t-1}\times Ku_t + V^{TSD}_{t-1}\times\psi^D + V^{TSE}_{t-1}\times\psi^E - D_{t-1}\times Kd_t \qquad (A4b)$$

Replacing Vu_{t-1} by it value

$$E_{t-1}\times Ke_t = \left(D_{t-1} + E_{t-1} - V^{TSD}_{t-1} - V^{TSE}_{t-1}\right)_t \times Ku_t + V^{TSD}_{t-1}\times\psi^D + V^{TSE}_{t-1}\times\psi^E - D_{t-1}\times Kd_t \qquad (4c)$$

Dividing by E_{t-1}

$$Ke_t = \left(\frac{D_{t-1}}{E_{t-1}} + 1 - \frac{V^{TSD}_{t-1}}{E_{t-1}} - \frac{V^{TSE}_{t-1}}{E_{t-1}}\right)\times Ku_t + \frac{V^{TSD}_{t-1}\times\psi^D}{E_{t-1}} + \frac{V^{TSE}_{t-1}\times\psi^E}{E_{t-1}} - \frac{D_{t-1}\times Kd_t}{E_{t-1}} \qquad (A4d)$$

Simplifying

$$Ke_t = Ku_t + \left(\frac{D_{t-1}}{E_{t-1}} - \frac{V^{TSD}_{t-1}}{E_{t-1}} - \frac{V^{TSE}_{t-1}}{E_{t-1}}\right)\times Ku_t + \frac{V^{TSD}_{t-1}\times\psi^D}{E_{t-1}} + \frac{V^{TSE}_{t-1}\times\psi^E}{E_{t-1}} - \frac{D_{t-1}\times Kd_t}{E_{t-1}} \qquad (A4e)$$

Grouping we find the general expression for Ke

$$Ke_t = Ku_t + (Ku_t - Kd_t)\times\frac{D_{t-1}}{E_{t-1}} - \left(Ku_t - \psi^D\right)\times\frac{V^{TSD}_{t-1}}{E_{t-1}} - \left(Ku_t - \psi^E\right)\times\frac{V^{TSE}_{t-1}}{E_{t-1}} \qquad (A4f)$$

We present selected values for the discount rate of the TS

- If $\psi^D = \psi^E = Ku$

$$Ke_t = Ku_t + (Ku_t - Kd_t)\times\frac{D_{t-1}}{E_{t-1}} \qquad (A5)$$

- If $\psi^D = \psi^E = Kd$

$$Ke_t = Ku_t + \left(Ku_t - Kd_t\right)\times\frac{D_{t-1}}{E_{t-1}} - \left(Ku_t - Kd\right)\times\frac{V^{TSD}_{t-1}}{E_{t-1}} - \left(Ku_t - Kd_t\right)\times\frac{V^{TSE}_{t-1}}{E_{t-1}} \qquad (A6)$$

$$Ke_t = Ku_t + \left(Ku_t - Kd_t\right)\times\left(\frac{D_{t-1}}{E_{t-1}} - \frac{V^{TSD}_{t-1}}{E_{t-1}} - \frac{V^{TSE}_{t-1}}{E_{t-1}}\right) \qquad (A7)$$

- As $V^{TSD} = TD_{t-1}$ in perpetuity when $\psi^D = \psi^E = Kd$

$$Ke_t = Ku_t + \left(Ku_t - Kd_t\right)\times\left(\frac{D_{t-1}}{E_{t-1}} - \frac{T\times D_{t-1}}{E_{t-1}} - \frac{V^{TSE}_{t-1}}{E_{t-1}}\right) \qquad (A8a)$$

$$Ke_t = Ku_t + \left(Ku_t - Kd_t\right)\times\left(1-T\right)\times\frac{D_{t-1}}{E_{t-1}} - \left(Ku_t - Kd_t\right)\times\frac{V^{TSE}_{t-1}}{E_{t-1}} \qquad (A8b)$$

- If $\psi^D = \psi^E = Kd = Kf$ and the interest rate for equity is Kf, the risk free rate, and Ei=EBV (book value of E) and EBV is the basis for calculating interest on equity then

$$Ke_t \;=Ku_t+\left(Ku_t\text{-}Kf_t\right)\times\frac{D_{t\text{-}1}}{E_{t\text{-}1}}\text{-}\left(Ku_t\ \text{-}Kf_t\right)\times\frac{T\times D_{t\text{-}1}}{E_{t\text{-}1}}\text{-}\left(Ku_t\ \text{-}Kf_t\right)\times\frac{T\times BVE_{t\text{-}1}}{E_{t\text{-}1}} \qquad \text{(A9a)}$$

$$Ke_t \;=Ku_t+\left(Ku_t\text{-}Kf_t\right)\times\left(\frac{D_{t\text{-}1}}{E_{t\text{-}1}}\text{-}\frac{T\times D_{t\text{-}1}}{E_{t\text{-}1}}\text{-}\frac{T\times BVE_{t\text{-}1}}{E_{t\text{-}1}}\right) \qquad \text{(A9b)}$$

- If Ei, the basis for calculating interest on equity is Ei=E (E = market value of equity)

$$Ke_t \;=Ku_t+\left(Ku_t\text{-}Kf_t\right)\times\left(\frac{D_{t\text{-}1}}{E_{t\text{-}1}}\text{-}\frac{T\times D_{t\text{-}1}}{E_{t\text{-}1}}\text{-}T\right) \qquad \text{(A10)}$$

- If ψ^D = Kd y ψ^E = Ke then

$$Ke_t=Ku_t+\left(Ku_t\text{-}Kd_t\right)\frac{D_{t\text{-}1}}{E_{t\text{-}1}}\text{-}\left(Ku_t\text{-}Kd\right)\frac{V^{TSD}_{t\text{-}1}}{E_{t\text{-}1}}\text{-}\left(Ku_t\text{-}Ke\right)\frac{V^{TSE}_{t.1}}{E_{t\text{-}1}} \qquad \text{(A11a)}$$

Solving for Ke

$$Ke_t\text{-}Ke\times\frac{V^{TSE}_{t.1}}{E_{t\text{-}1}}=Ku_t+\left(Ku_t\text{-}Kd_t\right)\frac{D_{t\text{-}1}}{E_{t\text{-}1}}\text{-}\left(Ku_t\text{-}Kd\right)\frac{V^{TSD}_{t\text{-}1}}{E_{t\text{-}1}}\text{-}Ku_t\times\frac{V^{TSE}_{t\text{-}1}}{E_{t\text{-}1}} \qquad \text{(A11b)}$$

Grouping

$$Ke_t\times\left(1-\frac{V^{TSE}_{t\text{-}1}}{E_{t\text{-}1}}\right)=Ku_t+\left(Ku_t\text{-}Kd_t\right)\frac{D_{t\text{-}1}}{E_{t\text{-}1}}\text{-}\left(Ku_t\text{-}Kd\right)\frac{V^{TSD}_{t\text{-}1}}{E_{t\text{-}1}}\text{-}Ku_t\times\frac{V^{TSE}_{t\text{-}1}}{E_{t\text{-}1}} \qquad \text{(A11c)}$$

Dividing by $\left(1-\frac{V^{TSE}_{t\text{-}1}}{E_{t\text{-}1}}\right)$ and simplifying

$$Ke_t=\frac{E_{t\text{-}1}\times Ku_t}{E_{t\text{-}1}\text{-}V^{TSE}_{t\text{-}1}}+\left(Ku_t\text{-}Kd_t\right)\frac{D_{t\text{-}1}}{E_{t\text{-}1}\text{-}V^{TSE}_{t\text{-}1}}\text{-}\left(Ku_t\text{-}Kd\right)\frac{V^{TSD}_{t\text{-}1}}{E_{t\text{-}1}\text{-}V^{TSE}_{t\text{-}1}}\text{-}Ku_t\times\frac{V^{TSE}_{t\text{-}1}}{E_{t\text{-}1}\text{-}V^{TSE}_{t\text{-}1}} \qquad \text{(A12a)}$$

Grouping and simplifying

$$Ke_t=Ku\times\frac{E_{t\text{-}1_t}}{E_{t\text{-}1}\text{-}V^{TSE}_{t\text{-}1}}\text{-}Ku_t\times\frac{V^{TSE}_{t\text{-}1}}{E_{t\text{-}1}\text{-}V^{TSE}_{t\text{-}1}}+\left(Ku_t\text{-}Kd_t\right)\frac{D_{t\text{-}1}}{E_{t\text{-}1}\text{-}V^{TSE}_{t\text{-}1}}\text{-}\left(Ku_t\text{-}Kd\right)\frac{V^{TSD}_{t\text{-}1}}{E_{t\text{-}1}\text{-}V^{TSE}_{t\text{-}1}} \qquad \text{(A12b)}$$

$$Ke_t=Ku_t+\left(Ku_t\text{-}Kd_t\right)\left[\frac{D_{t\text{-}1}\text{-}V^{TSD}_{t\text{-}1}}{E_{t\text{-}1}\text{-}V^{TSE}_{t\text{-}1}}\right] \qquad \text{(A12c)}$$

Traditional Textbook Formula for WACC for the FCF

The textbook formula has many restrictions and assumptions, for instance:

1. The only source of tax savings (TS) is interest on debt.
2. Taxes are paid the same period when accrued.
3. Existence of enough EBIT + Other income to earn the TS.

For the textbook formula for WACC the cash flows from the left hand side (assets) should be identical to the cash flows at the right hand side.

We depart from the basic M&M relations among CFs

$$FCF_t + TS_t^D + TS_t^E = CFD_t + CFE_t \quad (A13)$$

$$FCF_t = CFD_t + CFE_t - TS_t^D - TS_t^E \quad (A14)$$

For the FCF and WACC

$$V_t \times WACC_{t+1} = Kd_{t+1} \times D_t + Ke_{t+1} \times E_t - Kd_{t+1} \times D_t \times T - Kf_{t+1} \times E_t \times T \quad (A15a)$$

Solving for WACC

$$WACC_{t+1} = Kd_{t+1} \times (1-T) \times D\%_t + Ke_{t+1} \times E\%_t - Kf_{t+1} \times (1-T) \times E\%_t \quad (A15b)$$

The value of the firm is increased by the TS on interest on equity.

General WACC applied to the FCF

Formulating WACC in a general formulation eliminates some restrictions associated to the traditional textbook WACC, as mentioned above.

Let $WACC^{Gen}{}_i$ be the General WACC that is applied to the FCF in year i. We follow the same steps that we outlined for the standard WACC applied to the FCF and obtain an equation that is similar to equation 22.×

$$V^L{}_{t-1} \times WACC^{Gen}{}_t = D_{t-1} \times Kd_t - TS_t + E^L{}_{t-1} \times Ke_t \quad (A16a)$$

$$V^L{}_{t-1} \times WACC^{Gen}{}_t = V^{Un}{}_{t-1} \times Ku_t + V^{TS}{}_{t-1} \times \psi_t - TS_t \quad (A16b)$$

$$V^L{}_{t-1} \times WACC^{Gen}{}_t = (V^L{}_{t-1} - V^{TS}{}_{t-1}) \times Ku_t + V^{TS}{}_{t-1} \times \psi_t - TS_t \quad (A16c)$$

$$V^L{}_{t-1} \times WACC^{Gen}{}_t = V^L{}_{t-1} \times Ku_t - (Ku_t - \psi_t) \times V^{TS}{}_{t-1} - TS_t \quad (A16d)$$

Solving for the WACC in equation (A16d), we obtain,

$$WACC_t^{Gen} = Ku_t - (Ku_t - \psi_t) \times \frac{V_{t-1}^{TS}}{V_{t-1}^L} - \frac{TS_t}{V_{t-1}^L} \quad (A16f)$$

If we assume that the value of ψ_i is equal to the return to unlevered equity Ku_i, we can simplify equation (A16f) as follows.

$$WACC_t^{Gen} = Ku_t - \frac{TS_t}{V_{t-1}^L} \quad (A17)$$

Alternatively, if we assume that the value of ψi is equal to the cost of debt di, we can write equation (A16f) as follows.

$$WACC_t^{Gen} = Ku_t - (Ku_t - Kd_t) \times \frac{V_{t-1}^{TS}}{V_{t-1}^L} - \frac{TS_t}{V_{t-1}^L} \quad (A18)$$

This previous derivation is taken from Tham, J., and Vélez–Pareja I. (2004). *Principles of Cash Flow Valuation*, Academic Press.

We might think that ALL TS have the same discount rate, which is not too “elegant”. In that case we apply the previous formulation. The best and general approach is to work with a general formulation of WACC instead of the traditional textbook formulation that is specific for a particular case.

Repeating the procedure shown above, we have:

We depart from the basic equilibrium equations for cash flows and values:

$$FCF + TS = CFD + CFE \quad (A19)$$

$$V^{Un} + V^{TS} = D + E \quad (A20)$$

$$V^L_{t-1} \times WACC^{Gen}_t = D_{t-1} \times Kd_t - TS^D_t - TS^E_t + E^L_{t-1} \times Ke_t \quad (A21a)$$

Replacing the cash flows for D and E by the corresponding LHS cash flows we have:

$$V^L_{t-1} \times WACC^{Gen}_t = V^{Un}_{t-1} \times Ku_t + V^{TSD}_{t-1} \times \psi^D_t + V^{TSE}_{t-1} \times \psi^E_t - TS^D_t - TS^E_t \quad (A21b)$$

Replacing the unlevered value by the levered value minus the value of TS, we have

$$V^L_{t-1} \times WACC_{gen} = \left(V^L_{t-1} - V^{TSD}_{t-1} - V^{TSE}_{t-1}\right) \times Ku_t + V^{TSD}_{t-1} \times \psi^D_t + V^{TSE}_{t-1} \times \psi^E_t - TS^D_t - TS^E_t \quad (A21c)$$

Solving for WACC we obtain,

$$WACC^{Gen}_t = \left(\frac{V^L_{t-1}}{V^L_{t-1}} - \frac{V^{TSD}_{t-1}}{V^L_{t-1}} - \frac{V^{TSE}_{t-1}}{V^L_{t-1}}\right) \times Ku_t + \frac{V^{TSD}_{t-1}}{V^L_{t-1}} \times \psi^D_t + \frac{V^{TSE}_{t-1}}{V^L_{t-1}} \times \psi^E_t - \frac{TS^D_t}{V^L_{t-1}} - \frac{TS^E_t}{V^L_{t-1}} \quad (A21d)$$

Developing the term inside parenthesis, multiplying by Ku and grouping we have

$$WACC^{Gen}_t = Ku_t - Ku_t \times \frac{V^{TSD}_{t-1}}{V^L_{t-1}} - Ku_t \times \frac{V^{TSE}_{t-1}}{V^L_{t-1}} \times Ku_t + \frac{V^{TSD}_{t-1}}{V^L_{t-1}} \times \psi^D_t + \frac{V^{TSE}_{t-1}}{V^L_{t-1}} \times \psi^E_t - \frac{TS^D_t}{V^L_{t-1}} - \frac{TS^E_t}{V^L_{t-1}} \quad (A21e)$$

$$WACC^{Gen}_t = Ku_t - \left(Ku_t - \psi^D_t\right) \times \frac{V^{TSD}_{t-1}}{V^L_{t-1}} - \left(Ku_t - \psi^E_t\right) \times \frac{V^{TSE}_{t-1}}{V^L_{t-1}} - \frac{TS^D_t}{V^L_{t-1}} - \frac{TS^E_t}{V^L_{t-1}} \quad (A21f)$$

This is the general formulation for WACC for the FCF. Observe it has the same structure than the one developed above.

Now we define scenarios for the discount rate for the TS

- If $\psi^D = \psi^E = Ku$ then

$$WACC^{Gen}_t = Ku_t - \frac{TS^D_t}{V^L_{t-1}} - \frac{TS^E_t}{V^L_{t-1}} \quad (A22)$$

- If $\psi^D = \psi^E = Kd$ then

$$WACC^{Gen}_t = Ku_t - (Ku_t - Kd_t) \times \left[\frac{V^{TSD}_{t-1}}{V^L_{t-1}} + \frac{V^{TSE}_{t-1}}{V^L_{t-1}}\right] - \frac{TS^D_t}{V^L_{t-1}} - \frac{TS^E_t}{V^L_{t-1}} \quad (A23)$$

- If $\psi^D = Kd$ y $\psi^D = Ke$ then

$$WACC^{Gen}_t = Ku_t - (Ku_t - Kd_t) \times \frac{V^{TSD}_{t-1}}{V^L_{t-1}} - (Ku_t - Ke_t) \times \frac{V^{TSE}_{t-1}}{V^L_{t-1}} - \frac{TS^D_t}{V^L_{t-1}} - \frac{TS^E_t}{V^L_{t-1}} \quad (A24)$$

General WACC applied to the CCF

We know that the CCF is equal to the sum of the FCF and the TS.

$$CCF_i = FCF_i + TS_i \quad (A25)$$

Let $WACC^{Gen}_i$ be the general WACC applied to the CCF. We follow the same steps that we outlined for the standard WACC applied to the FCF.

$$V^L_{t-1} \times WACC^{Gen}_t = V^{Un}_{t-1} \times Ku_t + V^{TS}_{t-1} \times \psi_t \quad (A26a)$$

$$V^L_{t-1} \times WACC^{Gen}_t = V^L_{t-1} \times Ku_t - (Ku_t - \psi_t) \times V^{TS}_{t-1} \quad (A26b)$$

Solving for the WACC, we obtain,

$$WACC^{Gen}_t = Ku_t - \left(Ku_t - \psi_t\right) \times \frac{V^{TS}_{t-1}}{V^L_{t-1}} \quad (A26c)$$

If we assume that the value of ψ_i is equal to the return to unlevered equity Ku_i, we can simplify equation (A26c) as follows.

$$WACC^{Gen}_t = Ku_t \quad (A27)$$

If we assume that the value of ψ_i is equal to the cost of debt Kd_i, we can write equation (A26c) as follows.

$$WACC_t^{Gen} = Ku_t - (Ku_t - Kd_t) \times \frac{V_{t-1}^{TS}}{V_{t-1}^{L}} \quad (A28)$$

This previous derivation is taken from Tham, J., and Vélez–Pareja I. (2004). *Principles of Cash Flow Valuation*, Academic Press.

In the same vein, if we asume that all TS (interest on equity and interest on debt) have the same risk (the same discount rate) we use the previous formulation.

When we introduce the two different sources of TS with their specific risks, we have

$$WACC_t^{Gen} = Ku_t - \left(Ku_t - \psi_t^D\right) \times \frac{V_{t-1}^{TSD}}{V_{t-1}^{L}} - \left(Ku_t - \psi_t^E\right) \times \frac{V_{t-1}^{TSE}}{V_{t-1}^{L}} \quad (A29)$$

For different values of the discount rate for the TS

- If $\psi^D = \psi^E = Ku$ then

$$WACC_t^{Gen} = Ku_t \quad (A30)$$

- If $\psi^D = \psi^E = Kd$ then

$$WACC_t^{Gen} = Ku_t - (Ku_t - Kd_t) \times \left[\frac{V_{t-1}^{TSD}}{V_{t-1}^{L}} - \frac{V_{t-1}^{TSE}}{V_{t-1}^{L}}\right] \quad (A31)$$

- If $\psi^D = Kd$ y $\psi^E = Ke$ then

$$WACC_t^{Gen} = Ku_t - (Ku_t - Kd_t) \times \frac{V_{t-1}^{TSD}}{V_{t-1}^{L}} - (Ku_t - Ke_t) \times \frac{V_{t-1}^{TSE}}{V_{t-1}^{L}} \quad (A32)$$