An Introduction to the Cost of Capital

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Abstract

This chapter is devoted to the definition and application of the “cost of capital” concept to the valuation of cash flows from different points of view. We present an approach to estimate the cost of debt and general formulations for the cost of equity and the traditional weighted average cost of capital WACC, for the free cash flow, FCF and the non-traditional capital cash flow, CCF. We explain in detail the traditional textbook formula for the WACC with respect to the CCF and FCF.

We demonstrate the solution of the circularity problem between the WACC and the value of the cash flow.

At the end of the chapter we present some questions to encourage the reader to have further insights to the subject.

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JEL Classifications: E47, G12, G31, G30

An Introduction to the Cost of Capital

Introduction

To compare cash flows a firm has to define a cost of capital. This cost of capital will be used for discounting future cash flows to the present. We have to calculate the market value of the cash flow today and compare it with the amount invested.

When we use the expression “market value” we understand it to refer to the present value of future cash flows discounted at the average cost of capital. It is a proxy for what the stock market estimate for the traded firms. Precisely, these techniques of discounted cash flow (DCF) based on cash flows and the cost of capital are utilized in non traded firms that account for more than 99.5% of the firms in the world. This relationship between cash flows and cost of capital creates circularity: the cost of capital depends on value and value depends on cost of capital.

The resources for the firm come from two sources: the owners of equity and the owners of financial debt. In this chapter we study each source of funds in terms of their cost and the combined cost known as Weighted Average Cost of Capital, WACC.

Capital Markets

Firms obtain financing resources or opportunities for investment of excess cash in the capital and money markets.

A capital market is a place where investors and consumers of capital (generally companies or the government), raise long-term funds (longer than a year). Selling bonds and stocks are two ways to generate capital, thus bond markets and stock markets (such as the Dow Jones) are considered capital markets. The capital market is an effective and efficient mechanism for assigning and distributing the resources of capital in the process of transfering savings to investment (or the Circular Flow of Economic Activity).

The borrowing and lending of short-term obligations such as Treasury bills, commercial papers and bankers' acceptances occur in the money market.

Cost of Capital

The cost the firm pays for the resources that it must obtain to make the investments is not so evident. Here it is necessary to consider not only what is paid in terms of interest on a financial debt, but also what the shareholders *expect* to earn. In any case, firms pay for the use of funds from third parties and that price is the cost of capital.

Cost of Debt

In this context we call debt the financial debt. Financial debt is a liability that has a contractual interest rate and has to be paid in some period of time. It is not just any liability, but the one expected to generate interest charges.

Although in practice there is not a real distinction, we introduce a subtle but theoretically relevant difference: the market cost of debt and the contractual cost of debt. Market cost of debt is the discount rate the market uses to determine the value of a bond. This is the Internal Rate of Return, IRR, obtained when the future cash flows for the bond are compared with the price today. Contractual cost of debt is the rate of interest that is effectively used to calculate the interest charges. This distinction is of utmost importance because the former, market cost of debt, is used to estimate the value of debt and the later is used to calculate the tax savings as discussed in Vélez-Pareja and Tham (2010). There are other approaches to defining the market cost of debt: 1) To ask the lenders. 2) To estimate the grading of the firm issued by independent rating agencies such as Moody's, Fitch Investor Services or Standard and Poor's and using the Merrill Lynch Bond Index usually reported in The Wall Street Journal. However, current practitioners and firms use the contractual cost of debt as a proxy to the market cost of debt.

We assume in this chapter that the market cost of debt is identical to the contractual cost of debt. We also assume that the market value of debt is the book value of debt.

As mentioned above, one usual approach for estimating the cost of debt is to calculate the IRR of the future Cash Flow to Debt, CFD. We have to remind that IRR is an average that hides components inside the cost of debt, such as inflation. The cost of debt could change from period to period not only due to the inflation rate but also due to the composition of the debt portfolio of the firm.

Example 1

Assume a firm with total financing debt of $60 of which $10 are for one year at 14%; $40 for 5 years at 10%; and $10 for 3 years, at 19%. Interest rates and payments are on a yearly basis.

The traditional approach is to calculate a weighted average of the debt portfolio, and is shown in table 1, as follows:

Table 1 Weighting the Kd with different terms

Amount	Term	Kd	Weight	Kd
10	1 year	14%	16.7%	2.33%
40	5 years	10%	66.7%	6.67%
10	3 years	19%	16.7%	3.17%
Total average cost				12.17%

When using this approach we assume that the structure and proportion of the different sources of financing will be constant over time and that the firm will be financing its investment activities identically in the future.

In order to determine the cost of debt for a firm, the debt schedule of each loan has to be considered. It is not correct to make a weighting since the effect of the term of payment would not be considered. We should use the financial planning for the firm to estimate the cash flow of loans, CFL. The relevant information is the end of year balance and the interest payments. The combined cost per period is calculated as

$$Kd_t = \text{Interest charges}_t / \text{Ending Balance}_{t-1} \quad (1)$$

We continue with our example to clarify this idea. We construct the debt schedule for each loan, as follows. For loan 1 we have in table 2:

Table 2 Debt schedule 1

Year	Interest	Principal payment	Total payment	End of year Balance	Kd_1
0				10.0	
1	1.4	10.0	11.4	0.0	14%

For loan 2 we have in table 3:

Table 3 Debt schedule 2

Year	Interest	Principal payment	Total payment	End of year Balance	Kd_2
0				40.0	
1	4.0	6.6	10.6	33.4	10%
2	3.3	7.2	10.6	26.2	10%
3	2.6	7.9	10.6	18.3	10%
4	1.8	8.7	10.6	9.6	10%
5	1.0	9.6	10.6	0.0	10%

In this table we calculate the uniform payment at 5 years at a cost of 10% and we split that payment in two: the principal payment and the interest charge. As any payment is equal to the principal payment plus the interest charge, if we know the total payment (10.6), the interest rate 10% and the beginning balance we can calculate the interest and hence, the principal payment.

In the same vein, we have the debt schedule for loan 3 in table 4.

Table 4. Debt schedule 3

Year	Interest	Principal payment	Total payment	End of year Balance	Kd_3
0				10.0	
1	1.9	2.8	4.7	7.2	19%
2	1.4	3.3	4.7	3.9	19%
3	0.7	3.9	4.7	0.0	19%

In this table, we calculate uniform payment for 3 years at 19% and we split that payment into principal payment and interest charges.

The standard procedure in the best of cases is to combine the three schedules into one cash flow and calculate the IRR, as shown in table 5.

Table 5 Adding the three cash flows in one and calculating the IRR

Year	Loan 1	Loan 2	Loan 3	Total Loans
0	10	40	10	60.0
1	–11.4	–10.6	–4.7	–26.6
2		–10.6	–4.7	–15.2
3		–10.6	–4.7	–15.2
4		–10.6		–10.6
5		–10.6		–10.6
IRR				11.55%

The last column is simply the sum of the three payments. For instance, for year 1 we have −26.6 (−11.4 – 10.6 –4.7). Notice that IRR is 11.55% which means that the initial weighting was over estimating the average.

However, despite it being the most common approach, we consider that the relevant measure for the cost of debt is the combined yearly cost of debt calculated year by year. This is very important for considering the tax savings mentioned by Vélez-Pareja and Tham (2010). This is shown in table 6.

Table 6 Combined Debt schedule

Year	Interest	Principal payment	Total payment	End of year Balance	Kd
0				60.0	
1	7.3	19.3	26.6	40.7	12.17%
2	4.7	10.5	15.2	30.2	11.60%
3	3.4	11.9	15.2	18.3	11.17%
4	1.8	8.7	10.6	9.6	10.00%
5	1.0	9.6	10.6	0.0	10.00%

In the previous table we have calculated Kd for each year using (1). For instance, for year 1 we have 7.3/60 = 12.17%, and so on. We can observe in the previous table how the yearly interest rate varies from 10% up to 12.17%.

Observe how the calculation of averages, either the initial based on the amounts of loans or the weighted geometrical average resulting in the IRR, distort actual cost of debt that might change through the years. This change occurs because the proportion of each loan changes with time due to the term for each loan. However, it is not the only reason a Kd could change: when inflation rates change usually the cost of debt changes. This is what they call indexed rates that are quite common these days. This procedure we consider in eq. (1) is the correct one.

Under this approach, the cost of debt changes. It occurs because the proportion of each loan changes with time due to the term of payment for each loan. However, it is not the only reason a Kd could change: when inflation rates change usually the cost of debt changes. This is what they call indexed rates that are quite common these days. This procedure we consider in eq. (1) is the correct one.

The Cost of Equity, Ke: the Capital Asset Pricing Model

Here we only briefly mention the CAPM model because it is presented in detail in the other chapter; however, we present the model in a succinct form. This model says that the return of a stock is composed of a risk free rate and a risk premium that is a multiple of the equity or market risk premium. As a mathematical expression it says

(2)

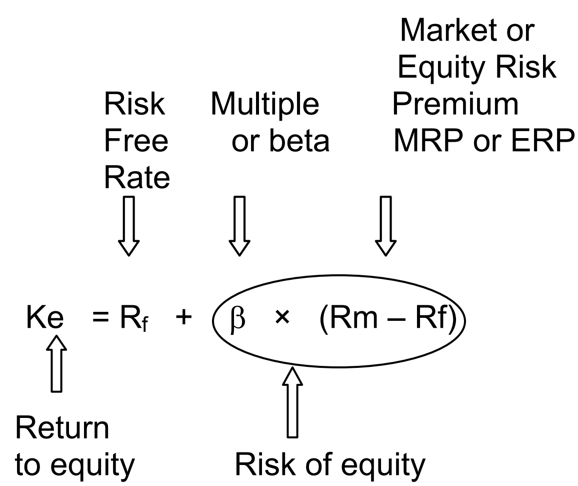

Exhibit 1. The CAPM model and its components

Differences between Kd and Ke

The firm receives funds from two sources: shareholders and debt-holders. The funds are a basket of funds which the firm uses to make investments. For the purpose of defining the cost of capital for a firm we distinguish between the cost of financial debt and the cost of equity.

Which is the difference between financial debt and equity? Debt is a source of funds regulated by a contract. On the other hand, equity has a residual return. This means that there is a difference in risk associated with each source of funds. As a consequence, the cost of equity is greater than the cost of debt.

Characteristics of Financial Debt:

1. It is regulated by a contract. The firm and the creditor define dates for paying back principal and interest charges.
2. The creditor receives her money without mattering if the firm has earnings or not.
3. The creditor has priority upon the payment of distributed dividends or net income.
4. Creditors require a warranty, usually based on tangible assets. Another requirement is to have cosigners to insure the debt payment.

Characteristics of Equity

1. Equity has a residual return. The firm will pay shareholders after it pays other creditors.
2. In case of bankruptcy or liquidation of the firm shareholders are the last to receive their money back.
3. The firm is not obliged to pay dividends.
4. The funds invested by shareholders have no term to be received back from the firm.

The previous characteristics show a great difference in the risk every source of funds bear.

In general, for the different rates the following relation holds, according to their risk level:

$$Ke > K_p > Ku > Kd > R_f \qquad (3)$$

where Ke is the cost of common stock, K_p is the cost of preferred stocks, Ku is the cost unlevered equity, Kd is the cost of debt, and R_f is the risk free rate. Therefore, the cost of capital is an intermediate rate between Ke and Kd.

Calculations for Ke and Ku

As there exists relationships between Ke, Kd and Ku once we have two of them we can calculate the third. Our preference is to estimate Ku and from there, estimate Ke. We will mention the procedures to estimate Ku and they can be used to estimate Ke as well.

This relationship between Ke, Kd and Ku is

$$Ku_t = Kd \times D\%_{t-1} + Ke \times E\%_{t-1} \qquad (4)$$

Where D% is the percentage of debt on total value and E% is the percentage of equity on total value. We will show this below. It is intuitively correct since (4) is the average cost of resources weighted by the proportion of each source in the total.

In order to estimate Ku, we have several alternatives:

1. Defining an unlevered beta for similar firms as a proxy to actual beta and averaging this unlevered beta using findings from Hamada (1969).

$$\beta_{nt} = \frac{\beta_{proxy}}{\left[1 + \frac{D_{proxy}}{E_{proxy}}\right]} \qquad (5)$$

Using this beta we can use CAPM to obtain Ku.

2. We could make the owner imagine a scenario of no debt and ask her for how much she is willing to earn assuming no debt.

3. Another way to estimate Ku is assessing subjectively *the risk for the firm* and this risk could be added to the risk free rate. Cotner and Fletcher, (2000) present a methodology to calculate the risk of a firm not publicly held based the Analytical Hierarchy Process developed by Saaty (1982).

This Ku is in accordance with the actual level of debt but it has to be remembered that Ku is, according to Modigliani and Miller, M&M, (1958, 1963), constant and independent from the capital structure. Ku is named as cost of assets or of unlevered equity

Circularity might arise if we wish to estimate Ke, when Ku is estimated directly. The relevance of estimating Ku is that using the Capital Cash Flow, CCF, and Ku we can calculate the value of future capital cash flows of the firm or project. In addition, no circularities will be present and there is no need to calculate the leverage ratio for every period.

The Calculation of the WACC.

In this section we show a way to understand and derive the traditional textbook formula for WACC.

The Modigliani-Miller Proposal

M&M (1958 and 1963) say that in an economy with no taxes, the firm value does not depend on how it decides to finance itself: That is, that with perfect market conditions, the value of the firm is independent of the capital structure. The capital structure of the firm is the combination of debt and equity in the financing strategy.

That is, V the value of the levered firm is equal to V^{UL} the value of the unlevered firm.

$$V = V^{UL} \qquad (6)$$

And in turn,

$$V = E + D \qquad (7)$$

The implication of this is that the cost of capital will remain constant no matter how the capital structure changes. To keep Ku constant, Ke must change with the amount of leverage (assuming that the cost of debt is constant).

Given that the cost Ku, is constant, Ke, the cost of equity changes according to the leverage. From (4), Ke is

$$Ke_t = Ku_t + (Ku_t - Kd) \times D\%_{t-1}/E\%_{t-1} \qquad (8)$$

What is the meaning of equation 8? Since Ku and Kd are constant, we see that the return to levered equity Ke is a linear function of the debt-equity ratio. It should be no surprise that there is a positive relationship between Ke, the return to levered equity and the debt-equity ratio. Since the debt-holder has a prior claim on the expected cash flow generated by the firm, the risk to the equity holder is higher and the equity holder demands a higher return to compensate for the higher risk. The higher the amount of debt, given a constant total value, the higher is the risk to the equity holder, who is a residual claimant.

Equation 8 shows the relationship between Ke, and the debt-equity ratio. The following table shows the relationship between D, D/E, E, and Ke.

This can be seen in table 7 and exhibit 2.

Table 7: Ke as a function of D and the debt-equity ratio

D	E	D/E	Ke
0	1000	0.00	15.1%
200	800	0.25	15.9%
400	600	0.67	17.2%
600	400	1.50	19.8%
800	200	4.00	27.5%
900	100	9.00	43.0%

If the amount of debt is 200, the debt-equity ratio is 0.25 and the return to levered equity is 15.9%. If the amount of debt increases from 200 to 400, the return to levered equity increases by 1.3%, from 15.9% to 17.2%. However, the relationship between Ke, and the amount of debt D is non-linear (E = V – D and D/E is D/(V-D)).

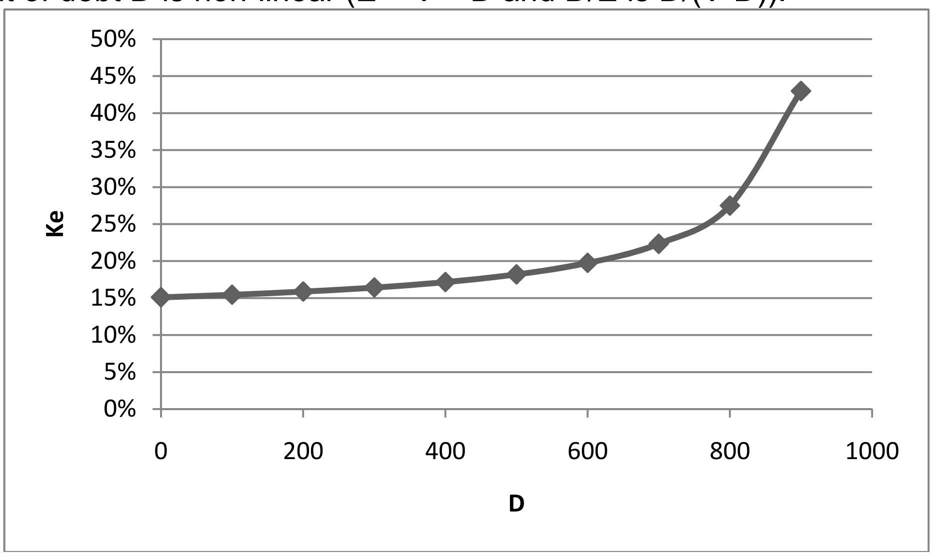

Exhibit 2. Ke as a function of D

Expressions for WACC

We distinguish between WACC for the Capital Cash Flow, CCF, $WACC^{CCF}$ and WACC for the Free Cash Flow, FCF, $WACC^{FCF}$. As we know the cost of debt and the cost of levered equity, we can estimate the average cost of capital for the capital cash flow, CCF, the $WACC^{CCF}$

WACC for the Capital Cash Flow, CCF

A simple approach to the $WACC^{CCF}$ is to think that owners of debt expect to receive the interest calculated with $D_{t.-1}$ and Kd. This is,

Interest = $Kd_t \times D_{t-1}$ (9a)

On the other hand, the share-holder expects to receive her expected Ke times the market value of equity. This is,

Expected return to equity = $Ke_t \times E_{t-1}$ (9b)

The sum of these two expectations is what the firm expects to pay for the funds supplied by these two “owners”; therefore if we wish to estimated the average cost we have to divide it by the value of those resources. This means the expected cost the firm will pay:

Total cost the firm expects to pay = $Kd_t \times D_{t-1} + Ke_t \times E_{t-1}$ (9c)

Cost of those funds as a percentage = $\frac{Kd_t \times D_{t-1} + Ke_t \times E_{t-1}}{D_{t-1} + E_{t-1}}$ (9d)

This is the weighted average for the CCF, $WACC^{CCF}$. The average cost of capital can be calculated as

$$WACC_t^{CCF} = \frac{Kd_t \times D_{t-1} + Ke_t \times E_{t-1}}{V_{t-1}} = Kd_t D\%_{t-1} + Ke_t E\%_{t-1} \quad (9e)$$

where $WACC^{CCF}$ is the average cost of capital for the CCF, and D% + E% equals 1.

This is exactly the definition of Ku :

$$WACC_t^{CCF} = Ku_t = Kd_t D\%_{t-1} + Ke_t E\%_{t-1} \quad (9f)$$

Note that (9f) is our equation (4). This has very useful and interesting implications. If we are able to estimate Ku, then we can avoid circularity that arises when working with $WACC^{FCF}$. We can calculate the firm value using the Capital Cash Flow, CCF and Ku. This makes the calculation of value a very easy task.

Let us examine if it makes sense that if the discount rate is the cost of capital then a project that returns exactly that cost of capital should have a NPV equal to zero. Next example is an answer to this question.

Example 2

Assume a project with an investment of $30 millions, of which 21 are financed by debt at 15.00%. The unlevered cost of equity, Ku is 18.84%. Corporate tax rate is 35%. The project has a life of one year. Table 8 shows the Income Statement.

Table 8 Income Statement

Sales revenue	70.00
Depreciation	30.00
Overhead and other expenses	33.00
Earnings Before Interest and Taxes EBIT	7.00
Interest charges	3.15
Earnings Before Taxes	3.85
Taxes	1.35
Net Income	2.50

If the project is liquidated at the end of the first year, the total funds distributed to debt and share-holders is 35.65, and is disaggregated as in table 9:

Table 9. Uses of generated cash in the project

Initial Investment	30.00
Distributed Net Income	2.50
Interest payments	3.15
Total	35.65

Table 10 shows the cash generated by depreciation will pay the investment made by each stakeholder, as follows

Table 10. Investment by source

Loan	21
Equity investment	9

Therefore, table 11 shows the cash flow to debt, CFD and cash flow to equity CFE, are:

Table 11. CFD and CFE

	Year 0	Year 1	Return
CFD	-21	24.2	15.00%
CFE	-9	11.5	27.81%

Hence, we have

CCF = CFD + CFE (10)

CCF = 24.2+ 11,5 = 35.7

When we discount the CCF with Ku we have

PV(CCF) = 35.7/1.1884 = 30

This means that the NPV is 0.

When we calculate Ke using eq (8) for year 1 using Ku we have

Ke_t = 18.84% + (18.84% − 15%) × 21/(30 − 21) = 27.81%

The other way around, if we calculate Ku using this Ke and eq. (9f) we find

Ku_t = 15% × 70% + 27.81% × 30% = 18.84%

A zero NPV means that the project just repays the initial investment and the cost of capital for each investor (debt and equity holder). Next we explain how this happens.

The debt-holder must receive the 21 millions she lent (70% × 30 = 21) plus interest of 15%, this is, 3.15 millions (15% × 21 = 3.15) and equity holders will receive when the project is liquidated (at the end of year 1), the 9 they invested, plus their expected return this is, 27.81% on their investment or what is the same, 27.81% × 9 = 2.5. All this amounts to 35.7 as shown above. This means that NPV = 0.

This is the simplest way to approach the problem of cost of capital.

Observe the symmetry between the components of $WACC^{CCF}$ and CCF. Each element of CCF has a corresponding element if the $WACC^{CCF}$. This is shown in table 12.

Table 12. Symmetry between cash flows and elements of $WACC^{CCF}$

	Debt	Equity
CCF	CFD	CFE
$WACC^{CCF}$	KdD%	KeE%

Example 3

Using the same example worked out by Vélez-Pareja and Tham (2010) for determining the cash flows, we can now value the firm with the Capital Cash Flow, CCF and using the "before" tax WACC or WACC for the CCF as shown in table 13.

Table 13 Calculation of total firm and equity value

Year	0	1	2	3	4
Observed Nominal Ku, cost of unlevered equity.	15.00%				
Inflation rate.	6.0%	6.0%	5.5%	5.5%	5.0%
Real Ku, ku	8.49%	8.49%	8.49%	8.49%	8.49%
Nominal Ku.	15.00%	15.00%	14.46%	14.46%	13.92%
CCF	-67.1	22.1	16.0	17.0	2.5
V	187.4	193.4	205.3	218.0	245.8
Debt	53.6	35.5	31.6	28.1	35.2
Equity. = V - D	133.7	157.9	173.7	189.9	210.6

We have included a terminal value (245.9) in year 4 for illustration and completeness purposes. The discussion of terminal value is beyond the scope of this chapter. We only say that that in this example it was calculated as a non growing perpetuity.

From this table we observe that we start with a "given" Ku (calculated using some of the procedures above mentioned). We deflate it with the inflation rate at instant 0. From there we forecast the nominal Ku inflating the real Ku with the forecasted inflation. For instance: ku= $\frac{1+Ku}{1+\pi}-1=\frac{1.15}{1.06}=8.49\%$ for instant 0 and we assume it is constant during the forecasting horizon. For year 1 and on we inflate ku as follows: $Ku_2=(1+ku)(1+\pi_2)-1=1.0849\times 1.055-1=14.46\%$.

With the forecasted Ku we can now discount the CCF and the terminal value. Remembering the basic tenet of finance we calculate the present value of the CCF as follows:

$$V_t=\frac{CF_{t+1}+V_{t+1}}{1+WACC^{CCF}_{t+1}} \qquad (11)$$

For year 4 we have

$$V_t=\frac{2.5+245.8}{1+13.92\%}=218.0$$

In a spreadsheet, the formula is copied down to left to obtain value at 0 as 187.4. For every year we obtain the forecasted firm value. Equity value is calculated solving for E in equation (7).

The Effect of Taxes on the Cost of Capital

In the previous section it might seem that we are disregarding taxes. We are not. Taxes are already taken into account in the Net Income distributed to share-holders. Now we will derive the cost of capital explicitly taking into account the effect of taxes.

When there are corporate taxes the idea proposed by M&M, (1958, 1963) is different. They proposed that when taxes exist the total value of the firm changes. This occurs because the government pays a subsidy for any deductible expense. This affects favorably the cash flow as fewer taxes are paid. In particular, this is true for interest payments. The value of the subsidy (the tax saving) is $T \times Kd \times D_{t-1}$. This was studied in Vélez-Pareja and Tham (2010).

Hence V increases by the value of the tax savings.

$$V = V^{UL} + V^{TS} = D + E \quad (12)$$

Associated to equation (12) there are correlated cash flows, as follows:

$$FCF + TS = CFD + CFE \quad (13)$$

Where TS is tax savings.

Equations 12 and 13 are the equilibrium equations for value and cash flows. Borrowing is something that is rejected intuitively by many investors and managers, especially in small and medium size enterprises. However, some degree of debt is good for the firm and for shareholders. Why firms do not finance 100% with debt? Because there are some costs associated to high leverage levels that deter the firm from doing that. These costs offset the TS and eventually put the firm in a condition of bankruptcy.

One of the key issues is the appropriate discount rate for the tax shield to obtain its value, V^{TS}. We assume that the correct discount rate for the tax shield is Ku.

The WACC for the Free Cash Flow, FCF

Most finance textbooks (see for instance Benninga and Sarig, 1997, Brealey, Myers and Marcus, 2004, Brealey, Myers and Allen, 2006, Copeland, Koller and Murrin, 1994, 2000, Damodaran, 1996, Gallagher and Andrew, 2000, Higgins, 2004, Palepu, Healy and Bernard, 2004, Ross, Westerfield and Jaffe, 2008, Van Horne, 2002, Weston and Copeland, 1992, Berk and Demarzo 2009) present the Weighted Average Cost of Capital WACC calculation as:

$$WACC = Kd \times (1-T) \times D\% + Ke \times E\% \quad (14)$$

Where T is the tax rate.

All of them precise that the values to calculate D% and E% are market values, however, it is not clear how to proceed to solve the circularity implied in this assert. They devote special space and effort to calculate Kd and Ke, but little effort is devoted to the correct calculation of market values.

Tax Savings

As mentioned in the previous section in order to introduce the effect of tax savings in $WACC^{FCF}$ we multiply, Kd times (1 − T). Which is the meaning of this factor (1−T)? As seen in Vélez-Pareja and Tham (2010), we introduced the tax savings as T times the interest expense and under some conditions this is reflected in the cost of debt, Kd as seen in Vélez-Pareja and Tham (2010) and Tham and Vélez Pareja, 2004b.

TS are effectively received when taxes are paid. Why is this so important? Because textbooks, practitioners and lecturers use the textbook formula as if it were the most common and general case and it is not. On the contrary, that popular formula is a very special case where some conditions have to be met. For instance, in a startup project the most common situation is that during the first years the firm incurs in losses and no TS is earned or depending on the tax law, firms could pay taxes even in part in advance and part the same year or even the next year.

Understanding the Popular WACC^{FCF} Formula

As we did for WACC^{CCF}, if it is correct to say that the firm *expects* to pay what the debt and equity holders *expect* to receive minus tax savings. This is what the firm really pays net of tax benefits.

Total cost the firm expects to pay minus tax savings, TC_{TS}

$$= Kd_t \times D_{t-1} + Ke_t \times E_{t-1} - TS_t \qquad (15a)$$

But TS_t is equal to $Kd_t \times D_{t-1} \times T$

Then

$$TC_{TS} = Kd_t \times D_{t-1} + Ke_t \times E_{t-1} - Kd_t \times D_{t-1} \times T \qquad (15b)$$

As before, we obtain the cost of those resources as a percentage dividing the net cost by the *market value* of invested capital and that is WACC^{FCF}.

$$WACC_t^{FCF} = \frac{Kd_tD_{t-1} + Ke_tE_{t-1} - Kd_tD_{t-1}T}{D_{t-1} + E_{t-1}} = \frac{Kd_tD_{t-1} + Ke_tE_{t-1} - Kd_tD_{t-1}T}{V_{t-1}} \qquad (15c)$$

This is the firm average cost of capital taking into account the tax savings and that we use to discount the FCF.

$$WACC_t^{FCF} = Kd_tD\%_{t-1}(1-T) + Ke_tE\%_{t-1} = Kd_tD\%_{t-1} + Ke_tE\%_{t-1} - TKd_tD\%_{t-1} \qquad (15d)$$

In table 14 observe the symmetry between the elements of WACC^{FCF} and FCF. Each term from FCF has a corresponding one in WACC^{FCF}.

Table 14. Symmetry between cash flows and elements of WACC^{FCF}

	Debt	Equity	Tax effects
FCF	CFD	CFE	TS
WACC^{FCF}	$Kd_tD\%_{t-1}$	$Ke_tE\%_{t-1}$	$T \times Kd_t \times D\%_{t-1}$

This is, we reduce CCF by the tax savings to arrive to FCF and at the same time, we reduce WACC^{FCF} by $T \times Kd_t \times D\%_{t-1}$ to arrive to WACC^{FCF}. If CCF is reduced by TS and we do not do a parallel action to the WACC^{CCF}, we would be double counting the tax effect in the firm or project value.

The popular textbook formula (15d) has some assumptions that not always are fulfilled. We wish to stress some relevant issues that are, but usually are disregarded:

1. Market value is the present value at WACC^{FCF} of the future cash flows and is calculated period by period.
2. D% and E% are calculated using market values for D, E and V at the beginning of period t (t-1), where the WACC^{FCF} belongs. This is, $D\%_{t-1}$ (D_{t-1}/V_{t-1}) and $E\%_{t-1}$ (E_{t-1}/V_{t-1}).
3. Kd×(1-T), the after tax cost of debt, implies that the tax payments are made the same instant when accrued.
4. Because of 1., 2. and changing inflation rates, WACC^{FCF} might change with time.
5. WACC^{FCF} calculation creates circularity. In order to calculate value it is necessary to calculate WACC^{FCF}, and to calculate WACC^{FCF}, we need value.
6. That we fully earn the tax savings in the same year as taxes are paid. This means that earnings before interest and taxes (EBIT) are greater than or equal to the interest charges.

7. The only sources of TS are interest charges. There might be other sources of TS, as mentioned in Vélez-Pareja and Tham (2010). (See Vélez-Pareja and Benavides, 2009, Vélez Pareja and Tham 2003 and Tham and Vélez Pareja 2004b).
8. Market value of debt is equal to its book value and hence the contractual cost of debt is identical to the market cost of debt.

All these conditions mean that the formula applies to a very restricted case. Notice that the TS are received when taxes are paid, not when they are accrued. Moreover, the firm could delay the payment of interest but pay the taxes and it is when taxes are paid when the TS are earned.

In the case the firm or project is a non taxed activity then Kd(1 − T) should be replaced by Kd. This happens when EBIT plus Other Income are negative and there are no losses carried forward. In the same way, if taxes are not paid the same year as accrued, the factor (1−T) does not include the tax effect on cash flows.

It is not strictly necessary that taxes are paid in a different year to the one they accrue them. This is very easily to understand: just change the forecasting period to month or quarters. Usually taxes are paid in only one period. Therefore, in most periods there are not tax payments.

Example 4

Assume the same project from example 2 that requires 30 millions. To calculate Ke we have to solve circularity because of (8) and (4).

For calculating Ke, E is the present value of FCF for year 1 at $WACC^{FCF}$ minus debt. This creates circularity. When solving the circularity we find that Ku is 18.84% (the same as in example 2, of course), Ke is 27.81%, (the same as in example 2, of course) and $WACC^{FCF}$ is 15.17% lower than Ke and Ku and higher than Kd as expected. $WACC^{FCF}$ is calculated with equation (15d). At the same time V is 30-

As in Example 2 if we assume that taxes are paid in the same year when accrued, then the after tax cost of debt will be 9.75% (15% × (1− 35%)). Hence, $WACC^{FCF}$ = 9.75% × 70% + 27.81% × 30% = 15.17%.

Tax savings are 1.1 (3.15 × 35%). The tax savings reduce the net interest payments to 2.05 (3.15 – 1.10) this is, 3.15 × (1 − 35%), in consequence as studied in Vélez-Pareja and Tham (2010), the FCF will be (3.15 + 30,00 + 2.50 – 1.1 = 34.55). In table 5 the 34.55 can be disaggregated as follows:

Table 15. Project’s FCF

Initial Investment	30.00
Distributed Net Income	2.50
Interest payments	3.15
Minus TS (3.15 × 35%)	-1.10
Free Cash Flow, FCF	34.55

Therefore, table 6 shows the net present value NPV is

Table 16. Project NPV

	Year 0	Year 1
FCF	-30	34.55
PV(FCF)	30	
NPV(15.17%)	0.00	

Observe that this is the same firm value we arrived in example 2, using the CCF and Ku as discount rate.

This FCF has been derived from (13).:

FCF = CFD + CFE − TS (16)

This is the easiest and error free procedure to calculate the FCF.

A project with NPV equal to 0 means that the cash flow exactly recovers the investment plus the cost of money. Next we explain how this occurs.

The debt-holder must receive \$21 (70% × 30 = 21) plus interest of 15%, this is, \$3.15 (15% × 21 = 3.15) and the share-holders will receive the investment of 9, plus their expected return; this is 27.81% on their investment 2.50 (27.81% × 9) and the government gives a subsidy of 1.1 as TS. Funds generated by depreciation charges will serve to payback the initial investment, as seen in table 17:

Table 17. Payback of investment to debt and equity

Loan	21
Equity investment	9

Observe that TS for interest expenses are not received by the debt-holder: they are received by the equity holder. Hence, CFD and CFE are shown in tale 18:

Table 18. CFD and CFE

	Year 0	Year 1	Return
CFD	-21	24.15	15.00%
CFE	-9	11.50	27.81%

As can be seen in this example, when a project has a NPV equal to zero, cash flows cover exactly the payment of the investment and the cost of capital.

A General Formulation for Ke, WACC^{CCF} and WACC^{FCF}

It can be shown that we can have a general formulations for Ke and WACC for each cash flow: CCF and FCF. (See Tham and Vélez-Pareja 2002 and 2004b):

$$Ke_t = Ku_t + (Ku_t - Kd_t)\frac{D_{t-1}}{E_{t-1}} - (Ku_t - \psi_t)\frac{V_t^{TS}}{E_{t-1}} \quad (17a)$$

where ψ is the discount rate (or risk) of TS and V^{TS} is the value of TS. Therefore, Ke will depend on the assumption we make for ψ.

For ψ equal to Ku, then (17a) simplifies to

$$Ke_T = Ku_T + (Ku_T - Kd_T)\frac{D_{T-1}}{E_{T-1}} \quad (17b)$$

For WACC^{FCF}

$$WACC_t^{Adj.FCF} = Ku_t - (Ku_t - \psi)\frac{V_{t-1}^{TS}}{V_{t-1}^{L}} - \frac{TS_t}{V_{t-1}^{L}} \quad (18a)$$

For ψ = Ku we have:

$$WACC_t^{Adj.FCF} = Ku_t - \frac{TS_t}{V_{t-1}^L} \quad (18b)$$

Where $WACC^{Adj.FCF}$ is the adjusted WACC that results when applying the general expression for WACC.

For the $WACC^{CCF}$

$$WACC_t^{Adj.CCF} = Ku_t - (Ku_t - \psi)\frac{V_{t-1}^{TS}}{V_{t-1}^L} \quad (19a)$$

For ψ = Ku we have:

$$WACC_t^{Adj.CCF} = Ku_t \quad (19b)$$

Calculating WACC^{FCF} and Firm Value

In this section we show a similar example as found in Tham and Vélez-Pareja (2004) and Vélez-Pareja and Tham (2009).

Example 5

Using the same example from above we show in table 19 the input data for calculating the WACC and the value together, as follows

Table 19. Input data from previous calculations

	0	1	2	3	4
Debt	53.6	35.5	31.6	28.1	35.2
Cost of debt, Kd		13.12%	12.61%	12.61%	12.10%
CFD	-53.6	25.2	8.3	7.5	-3.7
CFE	-13.5	-3.1	7.7	9.5	6.2
CCF = CFE + CFD.	-67.15	22.13	16.04	16.98	2.48
TS.		2.46	1.57	1.40	1.19
FCF=CCF - TS	-67.15	19.66	14.47	15.58	1.29
Ku.		15.00%	14.46%	14.46%	13.92%

For estimating the $WACC^{FCF}$ we estimate the debt and equity participation in the firm market value for each period and calculate the contribution of each to the $WACC^{FCF}$. We will construct each table, step by step, assuming first that $WACC^{FCF}$ is zero.

As illustrated with equation (18b) we have another option to calculate $WACC^{FCF}$. We call it the adjusted $WACC^{FCF}$. This formulation is most general than the traditional textbook formula and can handle situations where some assumptions exposed above are not met.

$$WACC_t^{adj.\ FCF} = Ku_t - \frac{TS_t}{V_{t-1}} \quad (18b)$$

As said, the first step is to calculate the value with an arbitrary value for WACC, for instance, zero. See this in table 20. Our table for WACC and Value will appear as

Table 20 WACC calculations (Temporary results)

Year	0	1	2	3	4
(1a) Value in t=PV(FCF @ WACC) $WACC^{FCF}$	296.84	277.18	262.71	247.13	245.84
(1b) Value in t=PV(FCF @ WACC) $WACC^{Adj.FCF}$	296.84	277.18	262.71	247.13	245.84
(8a) $WACC^{FCF}$					
(8b) $WACC^{adj.FCF}$ = Ku-TS_t/V_{t-1}					

We do this temporary calculation to avoid division through zero as can be seen below.

We use the already known formulation (11):

$$V_t = \frac{CF_{t+1}+V_{t+1}}{1+WACC_{t+1}} \qquad (11)$$

For instance, firm value at end of year 3 is (1.29+245.84)/(1+0%) = 247.13.

We use the same terminal value above when we calculated value using the CCF.

For year 2 it will be (247.13+ 15.58) /(1+0%) = 262.71 and so on for the other years.

Done this we can calculate temporary values for D%, E% and Ke.

Table 21. WACC calculation. Contribution of debt to WACC. (Temporary results)

Year	0	1	2	3	4
(2) Relative weight of debt D% (Debt balance at t-1)/Total value of firm at t-1)		18.1%	12.8%	12.0%	11.4%
(3) Cost of debt after taxes Kd×(1-T)		8.5%	8.2%	8.2%	7.9%
(4) Contribution of debt to WACC, Kd×(1-T)×D_{t-1}%		1.5%	1.0%	1.0%	0.9%

In the same vein we estimate the contribution of equity to $WACC^{FCF}$.

Table 22. WACC calculation. Contribution of equity to WACC. (Temporary results)

Year	0	1	2	3	4
(5) Relative weight of equity E% = (1-D%)		81.9%	87.2%	88.0%	88.6%
(6) Cost of equity Ke = (Ku_t – Kd)×$D\%_{t-1}$/ $E\%_{t-1}$		15.4%	14.7%	14.7%	14.1%
(7) Contribution of equity to WACC = $E\%_{t-1}$×Ke		12.6%	12.8%	12.9%	12.5%

At this point we set the spreadsheet to handle iterations as follows:

1. Select the Office Button at the top left and select Excel Options in Excel (2007).
2. Select Formula.
3. Enable Iterations.
4. Click Ok.

This procedure can be done at any moment before we calculate the WACC, starting the work on the spreadsheet or when Excel declares the presence of circularity. After these instructions are followed, we recommend that the last arithmetic operation be the sum of the debt and equity contribution to arrive to $WACC^{FCF}$ and at the same time to the calculation of value. This is shown in table 23

Table 23. WACC and value V, calculation. (Final results)

Year	0	1	2	3	4
(1a) V in t=PV(FCF @ $WACC^{FCF}$) $WACC^{FCF}$	187.4	193.4	205.3	218.0	245.8
(1b) V in t=PV(FCF @ $WACC^{FCF}$) $WACC^{AdjFCF}$	187.4	193.4	205.3	218.0	245.8
(2) Relative weight of debt D% (Debt balance at t-1)/Total value of firm at t-1)		28.6%	18.4%	15.4%	12.9%
(3) Cost of debt after taxes Kd×(1-T)		8.5%	8.2%	8.2%	7.9%
(4) Contribution of debt to $WACC^{FCF}$, Kd×(1-T)×D_{t-1}%		2.4%	1.5%	1.3%	1.0%
(5) Relative weight of equity E% = (1-D%)		71.4%	81.6%	84.6%	87.1%
(6) Cost of equity Ke = (Ku_t – Kd)×D%$_{t-1}$/ E%$_{t-1}$		15.8%	14.9%	14.8%	14.2%
(7) Contribution of equity to $WACC^{FCF}$ = E%$_{t-1}$×Ke		11.2%	12.1%	12.5%	12.4%
(8a) $WACC^{FCF}$		13.7%	13.6%	13.8%	13.4%
(8b) $WACC^{adj.FCF}$ = Ku-TS_t/V_{t-1}		13.7%	13.6%	13.8%	13.4%

Note that the cost of equity –Ke– is larger than Ku as expected, because Ku is the unlevered cost of equity. When there is debt necessarily Ke ends up being greater than Ku, because the risk of leverage. With these values it is possible to calculate the firm value for each period.

The reader has to realize that the values 13.4% and 13.8%, etc. are not calculated from the beginning because they depend on value and WACC depends on V. In table 24 we derive the market value of equity.

Table 24 Calculation of equity value from firm value

Year	0	1	2	3	4
V	187.4	193.4	205.3	218.0	245.8
D	53.6	35.5	31.6	28.1	35.2
E = V - D	133.74	157.9	173.7	189.9	210.6

Independent Calculation of Equity Value, E

When the present value of CFE at Ke, is calculated the same result is obtained. This means that the right discount rate to discount the CFE is Ke, and its discounted value is consistent with the value calculated with the FCF.

In table 24 we calculated the market value of equity using the firm market value. However, this is not an independent method because we use the values from other method. In order to calculate the market value of equity in an independent way we will use the same procedure utilized for the calculation of value with $WACC^{FCF}$. The difference is that we will calculate again the value of Ke and using CFE.

Table 25 with Ke equal to zero is temporary and the only purpose is to avoid a division through zero.

Table 25. Initial table to calculate the market equity value. (Temporary results)

Year	0	1	2	3	4
E	230.9	234.0	226.3	216.8	210.6
D	53.6	35.5	31.6	28.1	35.2
Ke = Ku + (Ku-Kd)$D\%_{t-1}/E\%_{t-1}$					

Activating iterations as explained above, we arrive to the correct value of equity. The final table for this calculation is as follows in table 26,

Table 26. Independent calculation of market equity value. (Final table)

Year	0	1	2	3	4
V of equity	133.7	157.9	173.7	189.9	210.6
Debt	53.6	35.5	31.6	28.1	35.2
Ke = Ku + (Ku-Kd)$D\%_{t-1}/E\%_{t-1}$		15.8%	14.9%	14.8%	14.2%

Observe that working independently we reach identical values for equity, total value and Ke.

The Adjusted Present Value, APV

The same result is reached working with the left hand side of the equilibrium equation for cash flows. This is, calculating the present value for the free cash flow at Ku, and adding up the present value of tax savings at the discount rate we have assumed for the TS, in this case, Ku. Myers (1974), proposed this simple approach and it is known as Adjusted Present Value, APV. Myers and all the finance textbooks teach that the discount rate for the TS should be the cost of debt. However, as seen in Vélez-Pareja and Tham (2010) on cash flows and Vélez-Pareja, (2010), the tax savings depend on EBIT. Hence, the risk associated to the tax savings is the same as the risk of the free cash flows rather than the cash flow to debt. Therefore, the discount rate should be Ku.

APV = V = PV(FCF @ Ku) + PV(TA @ ψ) (20a)

In the case of ψ = Kd as proposed by Myers (194) is,

APV = V = PV(FCF @ Ku) + PV(TA @ Kd) (20b)

When ψ is Ku, APV is equivalent to PV(CCF at Ku).

The calculation of APV with ψ = Ku is shown in table 27.

Table 27. Calculation of APV with ψ = Ku.

Year	0	1	2	3	4
PV(FCF @Ku)	182.43	190.13	203.15	216.94	245.84
PV(TS @ Ku)	4.95	3.23	2.13	1.04	
V	187.39	193.36	205.29	217.99	245.84

Calculating NPV

With V and the initial investment we can calculate the NPV as seen in table 28.

Table 28. Firm NPV calculations from firm and Equity investment

Year	0
V	187.39
Invested capital = Total assets - Current liabilities + ST debt	67.15
NPV_{firm}	120.24
E	133.74
Invested initial equity	13.50
NPV_{equity}	120.24

Observe as well that the NPV for the equity holder is the same as the NPV for the project (firm). This result is expected first because we assumed that the market value of debt is its book value and second by definition, NPV is what is left for equity holders after they receive their expected return.

All these calculations give identical results. See table 29. There exist other methodologies equally consistent. See Tham, and Vélez-Pareja, 2004b.

In this example,

Table 29. Comparison of values by different approaches

Method	Value	E = V - D	NPV
PV(FCF at $WACC_{t.}$)	187.39	133.74	120.24
PV(FCF at Ku) + PV(TS at Ku)	187.39	133.74	120.24
PV(CCF at Ku)	187.39	133.74	120.24
PV(CFE at Ke)		133.74	120.24

The value of equity is the price that the owners would sell their participation in the firm and in this case, is higher than the initial equity contribution.

When using Kd as the discount rate for the TS, we find a higher value and full consistency as we did with the assumption the discount rate for the TS is Ku (in this example). In short, ALL methods if properly done yield the same value. (See Tham and Vélez Pareja, 2004b and Vélez-Pareja and Burbano-Perez, 2010)

Summary and Conclusions

The misuse of WACC might be due to several reasons. One of them is that there have not been computing tools to solve the circularity problem when calculating WACC. Now it is possible and easy with the existence of spreadsheets. Not having these computing resources in the previous years, it was necessary to use simplifications such as calculating just one single discount rate or in the best of cases to use the book values in order to calculate the WACC.

Here a detailed methodology to calculate the WACC has been presented taken into account the market values in order to weigh Kd and Ke. By the same token a methodology based on the WACC before taxes Ku, constant (assuming stable macroeconomic variables, such as inflation) that does not depend on the capital structure of the firm has been presented.

The most difficult task is the estimation of Ku, or alternatively, the estimation of Ke. Here, a methodology to estimate those parameters is suggested. If it is possible to estimate Ku from the beginning, it will be possible to calculate the total and equity value independently from the capital structure of the firm, using the CCF approach or the Adjusted Present Value approach and discounting the tax savings at Ku.

In summary, the different methodologies presented to calculate the total value of the firm are consistent and yield identical values:

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Appendix
Solving the Circularity Problem without Iterations.

In a provisional working paper Felipe Mejía and Ignacio Vélez-Pareja have developed an approach to solve the circularity problem. Departing from the basic tenet of finance

$E_{t-1} = (E_t+CFE_t)/(1 + Ke)$ (1)

Solving for 1+Ke

$1 + Ke = E_t/E_{t-1} + CFE_t/E_{t-1}$ (2)

When ψ = Ku, then $Ke = Ku + D_{t-1}(Ku-Kd)/E_{t-1}$, hence

$1+Ku + D_{t-1}(Ku-Kd)/E_{t-1} = E_t/E_{t-1} + CFE_t/E_{t-1}$ (3)

Reorganizing terms

$1+Ku = E_t/E_{t-1} + CFE_t/E_{t-1} - D_{t-1}(Ku-Kd)/E_{t-1}$ (4)

Multiplying by E_{t-1}

$(1+Ku)E_{t-1}=E_t + CFE_t - D_{t-1}(Ku-Kd)$ (5)

Solving for E_{t-1}

$E_{t-1}=(E_t + CFE_t - D_{t-1}(Ku-Kd))/(1+Ku)$ (6)

E_{t-1} does not depend on Ke nor on E_{t-1}.

Using the example from the body of the chapter we calculate the market value of equity as follows:

Table A1. Solving Directly the Circularity Problem

Year	0	1	2	3	4
CFE		-3.06	7.70	9.47	6.18
D	53.65	35.49	31.63	28.11	35.21
Kd		13.12%	12.61%	12.61%	12.10%
Ku		15.00%	14.46%	14.46%	13.92%
E	133.74	157.87	173.66	189.88	210.63
BVE	13.50				
NPV_{equity}	120.24				
VL	187.39				
Invested capital =	67.15				
NPV_{equity}	120.24				

This is an identical result we obtained in tables 26 and 28 in the chapter.

Discussion Questions

1. Discuss why when the discount rate for the tax savings is Kd, the cost of debt, the value of the cash flow is higher than when it is Ku, the cost of unlevered equity.
2. When debt is public the market defines its value that is usually different from the book value of debt. Discuss what does this mean in terms of market cost of debt and contractual cost of debt. Hint: which rate should be used for calculating the value of debt and which to calculate the amount of tax savings.
3. Discuss why if book value of debt is identical to its book value the net present value, NPV, of equity and of project or firm are identical. What would happen with the NPV if they are not identical?