Calculating Tax Shields from Financial Expenses with Losses Carried Forward

Ignacio Vélez-Pareja
ivelez@unitecnologica.edu.co, nachovelez@gmail.com
Universidad Tecnológica de Bolívar
Department of Finance and International Business
Instituto de Estudios para el Desarrollo (IDE)
Cartagena, Colombia

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Abstract

When calculating the Weighted Average Cost of Capital (WACC), the well-known textbook formula includes tax shields with the (1-T) factor affecting the contribution of debt to WACC. In this work we develop a procedure for properly calculating tax shields including the case when Losses Carried Forward are allowed and there is Other Income. The proper calculation of tax shields is relevant because the value of tax shields might be a substantial part of firm value.
We show that tax shields depend on Earnings before Interest and Taxes and therefore the risk of tax shields is the risk of the free cash flow; this is the cost of unlevered equity.

Keywords

Weighted Average Cost of Capital, WACC, firm valuation, tax shields, tax savings, losses carried forward.

JEL codes

D61, G31, H43

Calculating Tax Shields from Financial Expenses with Losses Carried Forward[1]

Introduction

Modigliani and Miller, MM, (1958) proposed that in the absence of taxes capital structure does not matter regarding the value of the firm. This is true in an ideal and perfect market. One of the major market imperfections are taxes. When corporate taxes exist (and no personal taxes), the situation posited by MM is different. They proposed that when taxes exist the total value of the firm does change. This occurs because no matter how well managed is the firm, if it is a subject of taxes there exists what economists call an externality. When the firm deducts any expense, the government pays a subsidy or contribution for the expense. It results in less tax payments. This is true for financial expenses and particularly interest payments. The amount of the subsidy (the tax saving) is $T \times Kd \times D_{t-1}$, where T is the tax rate, Kd is the market cost of debt (assumed to be identical to the contractual cost of debt) and D is the market value of debt (assumed to be identical to its book value) at the end of previous period. These tax savings are captured in the textbook formula for the Weighted Average Cost of Capital, WACC, as follows

$$WACC_t = Kd_t \times \frac{D_{t-1}}{V_{t-1}} + Ke_t \times \frac{E_{t-1}}{V_{t-1}} - \frac{Kd_t \times D_{t-1} \times T}{V_{t-1}} = Kd_t \times (1 - T) \times D\%_{t-1} + Ke_t \times E\%_{t-1} \quad (1)$$

Where WACC is the weighted average cost of capital, Kd is the cost of debt, T is the corporate tax rate, D_{t-1} is the market debt value at t-1, Ke_t is the cost of levered equity, E_{t-1} is the market value of equity, V_{t-1} is the market value of the firm, and D% and E% are the weights of debt and equity in the cost of capital.

[1] The basic ideas for this work are taken from Vélez-Pareja, 2009 and have been reproduced.

The effect of tax savings is captured in the textbook formula with the (1-T) factor affecting the contribution of debt to the WACC. Tax shields are a strange mix of accounting and accrual related to WACC that relies on market values.

In this work we develop a procedure for properly calculating tax shields including the case where Losses Carried Forward are allowed and when there is Other Income. This is relevant because the value of tax shields might be a substantial part of value. We show that tax shields depend on Earnings before Interest and Taxes and therefore the risk of tax shields is the risk of the free cash flow; this is, the cost of unlevered equity.

The paper is organized as follows: In Section One we review the existing literature on calculation of tax shields and its value; in Section Two we explain what the tax shields are and illustrate how do they arise; in Section Three we show some special and typical cases when the firm is levered and summarize the conditions to totally or partially earn the tax shields and show that the textbook formula for WACC is a very special case; we develop a procedure for calculating tax shields; in Section Four we present a general formulation for WACC that includes tax shields and their value; In Section Five we conclude.

Section One. Literature review

There is a stream of literature where the task is to calculate the value of tax savings. However, less effort is given to examine how the tax savings are calculated. Most authors and textbooks calculate tax shields only from the interest payments. The general approach they use for calculating tax savings, tax shields, is the tax rate multiplied by interest payments. This is, the implicit assumption is that the only source of tax shields is interest payments and there is always enough profit to earn the tax savings. For instance see Arditti and Levy, 1977, Gonedes, 1981, Masulis, 1983, MacKie-Mason 1990, Arzac, 1996 and Liu, 2009.

On the other hand we find that firms obtain tax savings from other than tax shields on interest expenses and in some cases do not obtain the tax shields in the year they pay taxes. For instance, see Dammon and Senbet, 1998, Graham 2000 and Grabowski, 2009.

Graham, 2000, recognizes that "each marginal tax rate incorporates the effects of non-debt tax shields, tax-loss carrybacks, carryforwards, tax credits, the alternative minimum tax, and the probability that interest tax shields will be used in a given year" Graham, p. 1902. In Grabowski words:

> Do companies realize deductions at the statutory tax rate (get full benefit of interest tax deduction in the period in which the interest is paid)? Researchers have developed a simulated expected tax rate model that simulates taxable income into the future. This process has shown that many companies do not expect to pay the highest marginal rate for long periods of time. Because of tax loss carry-backs and carry-forwards and the cyclical nature of some industries, a substantial number of companies can expect a very low tax rate. Grabowski, 2009, p. 38.

On the other hand, the idea of not being "able to utilize all their interest deductions fully because of […] insufficient taxable income" (Cordes and Sheffrin, 1983, p. 95) is not an academic straw man. It is real and this situation has been recorded in several papers that intend to estimate the effective tax associated with interest expenses. See for example, Newbould, Chatfield and Anderson, 1992 (p.53), when they say that "the ability to use tax shields each year is forecast and excess shields are rolled forward until they can be used" and Kaplan and Ruback, 1995.

Tax shields are relevant because, as Fama, and French, 1998, put it, "good estimates of how the tax treatment of dividends and debt affects the cost of capital and firm value are a high priority for research in corporate finance" p. 819. In the same vein, Kemsley and

Nissim, 2002, estimated that value for the debt tax shields is approximately as high as 40 percent of debt balance and 10 percent of firm value, net of the personal tax disadvantage of debt. Graham, 2000, shows that firms derive significant tax savings from debt.

Having an algorithm for calculating tax shields is of interest of analysts when forecasting financial statements and cash flows to estimate values of firm and equity. We deal with this problem in Section Three.

Section Two. What Tax Shields Are?

Tax shields or tax savings, are a subsidy that the government gives to those who incur in deductible expenses. All deductible expenses are a source of tax savings. This is, labor payments, depreciation, inflation adjustments to equity, rent and any expense if they are deductible. As we discount cash flows with a discount rate that takes into account the sources of financing (debt and equity) we introduce the effect of tax savings in what we know as the Weighted Average Cost of Capital (WACC). For this reason we are especially interested in the financial expenses (mainly, interest payments).

We prefer to consider financial expenses as a general concept instead of interest expenses because financial expenses comprise interest, banking commissions, (in some countries) foreign exchange losses, adjustment of stock of capital when inflation adjustments are applied to the financial statements and interest on capital stocks as in Brazil (see, Vélez-Pareja and Tham, 2003 and Vélez-Pareja and Benavides, 2009). All these financial expenses are sources of tax savings and are not included in the textbook formula for WACC (1). This means that using this formula indiscriminately, results in an upward bias in the estimation of WACC and a downward one for firm value. Gilson, Hotchkiss and Ruback 2000, p. 49, recognize this fact: "Capital cash flows measure the cash available to all holders of capital and include the benefit of interest and other tax shields". (Underlining is ours). See also, Kemsley and Nissim, 2002.

As we will show, an after tax expense E_{at} is equal to the before tax expense, E_{bt} multiplied by (1-T) and the tax shields or tax savings are $E_{bt} \times T$ when there is enough income to offset the expense.

$$E_{at} = E_{bt} \times (1-T) \quad (2a)$$

This means that the subsidy the firm receives from the government (the tax shields) is

$$TS = E_{bt} \times T \quad (2b)$$

In the case of financial expenses (assuming that the only source of tax shields is interest payments) the tax shield is $T \times Kd \times D_{t-1}$ where all the variables have been defined above. The firm acquires the right to earn the tax shields if there are enough Earnings before Interest and Taxes, EBIT plus Other Income, OI, to offset the financial expenses. However, the firm actually *receives that subsidy* when *it pays its taxes*.

Let us explain all this with a simple example. Assume a firm with Sales revenue of 1,000, a Cost of goods sold, COGS of 500, Financial Expenses FE, of 200 and a corporate tax rate of 30%,

Table 1a. Income Statement (up to EBIT)

	No debt	With debt
Sales revenue	1,000	1,000
Cost of goods sold, COGS	500	500
EBIT + OI	500	500

What would happen if in the next line we include financial expenses? The first answer could be that Net Income, NI will be reduced by 200. However, as we show in Table 1b it is true for Earnings Before Taxes, EBT, as follows:

Table 1b. Income Statement (up to EBT)

	No debt	With debt
Sales revenue	1,000	1,000
COGS	500	500
EBIT + OI	500	500
FE	0	200
EBT.	500	300

However, Table 1b is not the complete picture. We have to see what happens with Net Income when taxes are included in the analysis.

Table 1c. Complete Income Statement

	No debt	With debt
Sales revenue	1,000	1,000
COGS	500	500
EBIT + OI	500	500
FE	0	200
EBT.	500	300
Taxes (30%)	150	90
Net income, NI	350	210

If financial expenses increased by 200 (from 0 to 200) the first reaction was to think that Net Income would be reduced by 200. What we found in the complete picture is that NI is not reduced by 200 but by 140. Using (2a) we have

$E_{bt} \times (1\text{-}T) = 200 \times (1\text{-}30\%) = 140$

Observe that the difference in taxes is the same as the difference between E_{at} and E_{bt} as expected. In fact, the tax savings are identical to the difference in taxes (150 – 90 = 60). And at the same time, that difference is, using (2b), $E_{bt} \times T$ or $200 \times 30\% = 60$. Later we will eliminate this constraint.

Section Three. Special and Typical Cases: A Simple Algorithm for Tax Shields Calculation

There are cases where the firm does not earn the full tax shields. The reader will easily identify situations where the previous conditions are not met, such as start-ups and firms in financial distress. Let us consider two situations:

1. Tax shields when EBIT+OI ≥ FE
2. Tax shields when 0 ≤ EBIT+OI<FE

The question is if it makes any difference. It does. We will show this in the next simple example.

Before moving forward let us consider the following assertion: The "right" to earn the tax shields is based on results from the Income Statement, IS and this financial statement is based on accruals that do not imply a cash flow. This means that the tax shields are based on accounting figures. In fact, any one could verify that taxes are calculated on the basis of accounting results that imply accruals. As can be seen from the IS, the "right" to the tax shields depends on EBIT and OI.

In the next table we examine the first case. We will use the same idea of a levered and unlevered firm that is subject to income taxes.

Table 2a. Tax shields when EBIT+OI ≥ FE

	No debt	With debt
EBIT + OI	200	200
FE	0	150
EBT	200	50
Taxes 40%	80	20
Net Income	120	30
TS = difference in taxes	0	60

We increased FE from 0 to 150. Net Income is reduced from 120 to 30. The net reduction was 90. And the tax shields were 60, this is 150 × 40% = 60, according to eq. (2b). Usually, when EBT is negative or zero, the firm pays no income taxes[2]. From the previous case we have

Proposition 1. When EBIT+OI ≥ FE tax shields are the corporate tax rate multiplied by the financial expenses, T×FE.

In table 2b we show the case when 0 ≤ EBIT+OI<FE.

Table 2b. Tax shields when 0 ≤ EBIT+OI<FE

	No debt	With debt
EBIT + OI	100	100
FE	0	150
EBT	100	-50
Taxes 40%	40	0
Net Income	60	-50
TS = difference in taxes	0	40

Observe that in this case EBIT is 100 and we increased FE from 0 to 150, BUT EBIT + OI< FE. Net Income was reduced from 60 to -50. The net reduction was 110. And the tax shields were 40. This 40 IS NOT 150× 40% = 60 as predicted by eq. (2b). This means that our rules (2a) and (2b) do not apply! It is interesting to observe that the firm is subject to income tax BUT it does not pay taxes in this case. Taxes are zero and yet the firm earns tax shields of 40. From this case we have,

Proposition 2. When 0 ≤ EBIT+OI < FE, tax shields are not corporate tax rate multiplied by financial expenses. On the contrary, tax shields are corporate tax rate multiplied by EBIT plus Other Income.

And

Proposition 3. When EBIT+OI < 0, tax shields are zero.

[2] We are not considering here a situation where the firm is subject to presumptive taxation. However, this type of taxes is applied precisely when the firm has EBT ≤ 0, hence the TS do not occur.

We can summarize these ideas in the following tables. Table 3a shows the case when EBIT+OI ≥ FE.

Table 3a. Case 1 EBIT+OI ≥ FE

No debt	Debt	Tax Shields = difference in taxes
EBIT + OI	EBIT + OI	
0	FE	
EBT = EBIT + OI	EBT = EBIT + OI- FE	
Tax = T × (EBIT + OI)	Tax = T × (EBIT + OI - FE)	TS = T×FE

In the previous case we say that the tax shields are equal to the financial expense multiplied by the tax rate. This is the case where the traditional textbook formula works. Now, in table 3b, we show the tax shields calculations for the case when EBIT + OI is non negative and less than FE.

Table 3b. Case 2 0 ≤EBIT+OI <FE

No debt	Debt	Tax Shields = difference in taxes
EBIT + OI	EBIT + OI	
0	FE	
EBT = EBIT + OI	EBT = EBIT+OI – FE < 0	
Tax = T × (EBIT + OI)	Tax = 0	TS = T × (EBIT + OI)

In this second case we see that the tax shields ARE NOT T multiplied by the financial expenses, as predicted by eq. (2b) but T multiplied by EBIT + OI. This is a very important conclusion. In table 3c we show the case when EBIT+OI < 0.

Table 3c. Case 3 EBIT+OI < 0

No debt	Debt	Tax Shields = difference in taxes
EBIT + OI	EBIT + OI	
0	FE	
EBT = EBIT + OI < 0	EBT = EBIT + OI - FE <0	
Tax = 0	Tax = 0	TS = 0

In this third case we say that when EBIT + OI is negative, the tax shields are zero. Observe that it is not true that tax shields arise if the firm actually pays taxes. Tax shields arise when the firm is subject to income taxes even if in a particular case the firm does not pay taxes, as in table 3b. The conditions for the existence of tax shields are that the firm is subject of income taxes AND EBIT + OI is non negative.

In summary, we have

$$TS=\begin{cases} T\times FE \text{ if EBIT+OI} \geq FE \\ T\times(\text{EBIT+OI}) \text{ if } 0 \leq \text{EBIT+OI} < FE \\ 0 \text{ if EBIT+OI} < 0 \end{cases} \quad (3)$$

What we have is a segmented function of tax shields depending from EBIT + OI. This is depicted in Exhibit 1.

It is interesting to note that Wrightsman, 1978, proposed this idea (see pp. 651-652). Surprisingly, because that was written more than 30 years ago and yet, we find most textbooks and papers assuming as a general rule that tax shields are tax rate multiplied by interest charges. This has implications because the use of the WACC textbook formula is generalized as if that were correct in all cases and in fact, it is a very special case. Observe what Cooper and Franks, 1983, p.572-573, assert:

> "With taxes as the only imperfection, no corporation pays taxes if it issues sufficient corporate debt to make interest charges always equal to taxable income from operations. Interest charges provide a costless alternative mechanism for sheltering taxable income, so alternative tax shield substitutes have no value. The capital budgeting rule requires that all projects should be evaluated on the basis of their pre-tax cash flows, using an unlevered equity required return as the discount rate".

This is not true. If EBIT exactly offsets interest charges, then taxable income is zero, BUT the tax shields are corporate tax rate multiplied by financial expenses according to (3). Hence, the discount rate has to include the effect of tax shields and should not be evaluated using only Ku, the unlevered cost of equity as Cooper and Franks, 1983 assert.

On the other hand, Aivazian and Berkowitz, 1992 suggest that the firm obtains tax shields when the firm pays taxes. As seen above, the firm could not pay taxes and still earns some tax shields. See the second segment of equation (3). When the firm is in that second segment it does not pay taxes and yet get some tax shields equal to T×(EBIT + OI).

Miles, and Ezzell, 1980, consider that tax shields cannot be known with certainty. This is obvious because tax shields depend on forecasts and on EBIT plus OI as noted above. However, what is true is that if we assume that tax shields are the resulting tax savings from interest payments and we use the textbook formula for WACC, tax savings will be biased.

This segmented function (3) is very important because in reality, at least in the beginning years for new ventures or startups, EBIT+OI might be less than FE or even negative. This means that in those cases we cannot assume that the tax shields are fully earned. It might be either zero or less than T×FE. Only when EBIT+OI is smaller than or equal to zero the firm earns no tax shields and in this case, the evaluation should be done used the unlevered cost of equity as discount rate. See Exhibit 1 and equation (3).

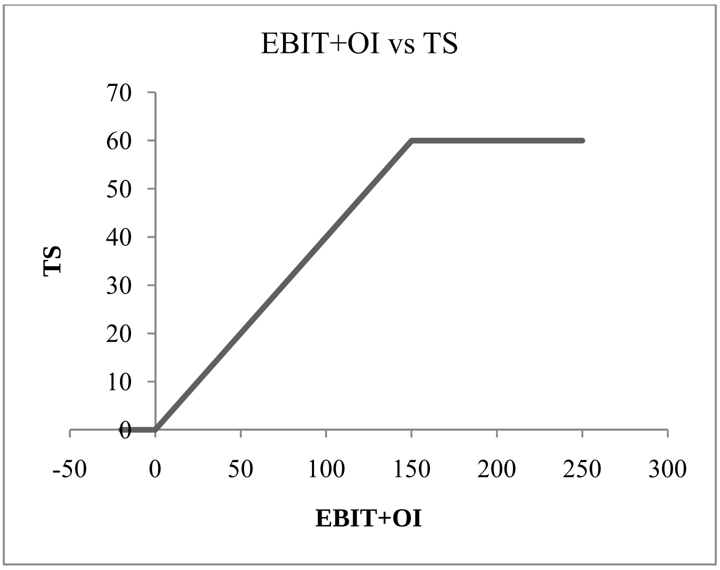

Exhibit 1. Tax shields as a function of EBIT + OI

The segmented function for tax shields can be expressed as

TS = Maximum(T × Minimum(EBIT+OI, FE), 0) (4a)

In Excel notation:

=Max(T*Min(EBIT+OI,FE),0) (4b)

If tables 3a to 3b and Exhibit 1 show the relationship between tax shields and EBIT, we can easily conclude that tax shields are a function of EBIT and hence have the same risk of EBIT (or Free Cash Flow). Wrightsman, 1978, calls this situation as risky debt; however, we consider that the riskiness of tax shields comes from EBIT, not from the debt itself. Following this we consider that this means the proper discount rate for tax shields should be Ku, the cost of unlevered equity. Regarding the risk of tax shields, Brennan and Schwartz, comments citing the seminal paper by Modigliani and Miller, 1963:

> This paper is concerned mainly with relaxing the assumption that the tax savings due to debt issuance constitute a "sure stream." Modigliani and Miller themselves acknowledge that "some uncertainty attaches even to the tax savings, though, of course, it is of a different kind and order from that attaching to the stream generated by the assets" (1963, n. 5). They attribute this uncertainty to two causes: first, the possibility of future changes in the tax rate and, second, the possibility that at some future date the firm may have no taxable income against which the interest payments on the debt may be offset. Brennan and Schwartz, 1978, p. 104.

If we examine textbook formula for WACC in (1) we have

$$WACC_t = Kd \times D\%_{t-1} \times (1-T) + Ke \times E\%_{t-1} \quad (1)$$

But it only applies to case 1. This is, if EBIT + OI ≥ 0, taxes are paid the same period when interest payments are the only source of tax shields.

Now we will show how the (1-T) factor works. Assume a loan of 1,000 to be repaid next year. Tax rate is 40% and taxes are paid the same year as accrued. The Cash Flow of Loan, CFL, is shown in the next table.

Table 4a. Taxes paid the same year. Tax shields fully earned.

Year	CFL[3]	TS	After tax CFL
0	1,000		1,000
1	-1,300	120	-1,180
IRR, Internal Rate of Return for the loan (from the firm's point of view)	30%		18%

If tax rate T is 40%, then tax shields are 120. In the previous table we have taxes paid the same year and the tax shields fully earned in the same year. In that case, after tax contractual Kd is Kd(1-T) = 30% × 60% = 18% which is equal to the IRR for the after tax CFL.

Now assume that taxes are paid next year. This means that tax shields are effectively received when taxes are paid. In that case we have:

Table 4b. Taxes paid next year. Tax shields fully earned.

Year	CFL	TS	After tax CFL
0	1,000		1,000
1	-1,300		-1,300
2		120	120
IRR for the loan (from the firm's point of view)	30%		20%

In table 4b we can see that after tax contractual Kd is not $Kd_{t-1}(1-T)$. Observe that postponing the tax shields one year increases the after tax cost of debt from 18% to 20%. This is of utmost importance because the trend is to use (1) as a standard and general formula when in fact, it is not. On the contrary, (1) is a very special case where some conditions have to be met. These conditions are:

1. Kd×(1-T) implies taxes are paid the same period when accrued.

[3] Note that CFL is the negative of the Cash Flow to Debt, CFD (CFD would be -1,000 in time 0 and +1,300 in time 1). However, after tax CFL IS NOT after tax CFD. The TS is not something that reduces the CFD (that is what the debt holder receives), it is a reduction of what the firm pays out. In fact, TS is a cash flow that goes directly to the equity holders. The meaning of after tax IRR refers not to the after tax cost of debt (as perceived by the debt holder), but to the net cost paid by the firm. (I am indebted to my colleague Rauf Ibragimov for calling my attention to clarify this).

2. Earnings before Interest and Taxes, EBIT plus Other Income, OI are greater than or equal to financial expenses and hence, the firm earns the full tax shields.
3. The only source of tax shields is the interest paid.
4. Market value of debt is equal to its book value and the contractual cost of debt is identical to the market cost of debt.

All these conditions mean that the formula applies to a very special and unique case. Also notice from table 4b that the tax savings are received when taxes are paid, not when they are accrued. Moreover, the firm could delay the payment of interest but pay the taxes and it is when taxes are paid when the tax shields are earned.

Proposition 4. Textbook WACC formula is valid only when conditions 1 to 4 hold.

Reality is not as simple as we have shown in the illustrative examples. Typical conditions in reality make the calculation and the "receipt" of tax shields a little more complex. Issues like inflation adjustment to the financial statements, losses carried forward, taxes paid in advance or delayed and exchange rate losses might make the calculation of tax shields difficult because in the financial model it is necessary to keep control of several conditions at a time and not all of them are reflected in the cost of debt after taxes, Kd×(1-T).

If Losses Carried Forward LCF, are allowed, tax shields not earned one period can be recovered in the future when losses are recovered. In table 5 we continue with the previous example in table 2b to illustrate the use of losses carried forward, LCF.

Table 5. Tax Savings with LCF from Year t to t+1

	Year t		Year t+1	
	No debt	With debt	No debt	With debt
EBIT	100	100	250	250
FE	0	150	0	150
EBT	100	-50	250	100
LCF				-50
Adjusted EBT = EBT + LCF	100	-50	250	50
Taxes 40%	40	0	100	20
Net Income = EBT - Tax	60	-50	150	80
TS = difference in taxes	0	40	0	80
TS	0	40	0	80
TS from FE	0	60	0	60
TS from LCF	0	0	0	20

Note that the effective tax rate in year t+1 with debt is 20% (20/100) instead of 40% as could be expected.

This means that our equations 4a and 4b are transformed into

TS = Maximum(T × Minimum(EBIT+OI, FE-LCF), 0) (5a)

In Excel notation:

=Max(T*Min(EBIT+OI,FE-LCF),0) (5b)

Applying equation 5a to the example in table 5 we have

TS = Maximum(T × Minimum(EBIT+OI, FE-LCF), 0)

= Maximum(40% × Minimum(250, 150+50), 0) = 80

This tax shields of 80 are decomposed, as shown in table 5, in 60 from FE (40%×150) and 20 from LCF (40%×50). Observe that tax shields are no longer T×FE. This means that the standard textbook formula for WACC, equation (1), is not longer valid in cases like the one shown in tables 2b and 5. Notice that the effect of larger tax shields in year t+1 from table 5 is not even captured by the "effective" tax rate. As T (effective) is introduced in (1), the effect is to raise the after tax cost of debt, instead of lowering it due to greater tax shields.

Proposition 5. When losses carried forward, LCF, are allowed and EBIT+OI ≥ FE Proposition 1 is modified to tax shields are the corporate tax rate multiplied by the financial expenses minus losses carried forward, T×(FE – LCF) if FE – LCF is not greater than EBIT + OI . The upper limit for tax shields in this case is T×(EBIT+OI).

When there is Other Income, the analysis of tax shields in terms of differences in taxes distorts the results. In that case we assume that an unlevered firm has Other Income and when levered, the other income or part of it, shifts to the payment of interest on debt. This is shown in table 6.

Table 6. Tax shields when the firm has Other Income and LCF

	No debt	With debt
EBIT	100	100
OI	100	50
EBIT+OI	200	150
FE		50
EBT	200	100
Taxes 40%	80	40
Net Income	120	60
TS	0	20

Using (4a) we have

$$TS = \text{Maximum}(T \times \text{Minimum}(EBIT+OI, FE), 0) \quad (4a)$$

$$TS = \text{Maximum}(40\% \times \text{Minimum}(150, 50), 0) = 20$$

Difference in taxes is 40. The reconciliation of these two figures is as follows: part of the difference in taxes comes from the fact that the levered firm has less Other Income to be taxed. In this case, that amount is 50. This means that the levered firm will have less taxes by the amount of 20 (40%×50). On the other hand, the firm has financial expenses by 50. From this amount, as EBIT + OI is greater than FE, hence the TS is 20 (40%×50). The difference in taxes is 40, but the TS is 20. From this example we see that difference in taxes is not identical to tax shields.

From the previous example we have

Proposition 6. When there is Other Income, the analysis of the levered and unlevered firm to define tax shields as the difference between taxes for unlevered and levered firm is distorted. Tax shields have to be calculated as stated in Propositions 1, 2 and 3 and not as differences in taxes. Difference in taxes includes the reduction of taxes when funds from Other Income are devoted to pay financial expenses.

And

Proposition 7. Difference in taxes has to be adjusted by $-T\times(OI_{unlevered} - OI_{levered})$.

From this proposition, the difference in taxes for the previous example (40) has to be adjusted by –40%(100-50) = -20. When we adjust the difference in taxes by -20 we obtain the correct tax shields.

Section Four. If Textbook Formula is a Special Case, is there a General Formulation for WACC?

Yes. There is a general formulation for WACC. Before we proceed we have to mention that the value of the tax shields is its present value at a proper discount rate, ψ. It can be shown that a general formulation for WACC for the FCF is (see Tham and Vélez-Pareja, 2002, 2004):

$$WACC_t = Ku_t - \frac{TS_t}{V_{i-1}} - (Ku_t - \psi_t)\frac{V^{TS}_{t-1}}{V_{t-1}} \tag{6a}$$

Where V^{TS} is the value of tax shields at ψ, Ku is the cost of unlevered equity and other variables have been defined previously. When we write WACC as a function of tax shields and its value as in (6a), tax shields could be earned at any time and from any source that affects the financing of the firm. It seems a complex formula, but it greatly facilitates work when working with WACC and the FCF, that is what we need in order to calculate value. For instance, it might be valid for cases where adjustment of equity capital is adjusted by inflation as in the case of financial statements adjusted by inflation (see Vélez-Pareja and Tham, 2003). It also might apply as said above, when part of dividends to shareholders are paid as interest on book value of equity, as in Brazil (see Vélez-Pareja and Benavides, 2009).

Depending on the assumption we make regarding the risk (the discount rate) of tax shields ψ, the expression for WACC is more or less complicated.

If we assume ψ = Kd

$$WACC_t = Ku_t - \frac{TS_t}{V_{t-1}} - (Ku_t - Kd_t) \times \frac{V^{TS}_{t-1}}{V_{t-1}} \quad (6b)$$

In this formulation we keep assuming that market cost of debt is identical to contractual cost of debt.

If we assume ψ = Ku

$$WACC_t = Ku_t - \frac{TS_t}{V_{t-1}} \quad (6c)$$

As can be seen in the previous formulas tax savings are included explicitly in the formulation for WACC. This means that we can introduce all types of tax savings as suggested in Section Two.

Section Five. Concluding Remarks

We have shown an algorithm to calculate tax shields with and without losses carried forward and how to use it in WACC formulation. In addition we showed that the condition for a firm to earn tax shields is not that it pays taxes; it is that the firm be subject to taxes

and that EBIT + OI be positive. This algorithm applies to all cases regarding the relationship between EBIT + OI and FE.

As was shown above, when there is OI difference in taxes for an unlevered and levered firm are not identical to tax shields. There the need to make adjustments taking into account the difference between Other Income between the levered and unlevered firm.

We also have shown that tax shields depend on Earnings before Interest and Taxes, EBIT and Other Income, OI. This means that a proper discount rate for tax shields is the cost of unlevered equity, Ku.

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