Will the Deflated WACC Please Stand Up? And the Real WACC Should Sit Down.

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First Version: March 10, 2010
This version: May 29, 2010

Abstract

In a world with taxes, there is a small discrepancy between the deflated WACC $WACC^{Def}$ and the real wacc. This is due to the (1-T) term that is in the standard expression for the WACC applied to the Free Cash Flow (FCF).

We compare different approaches for valuing nominal and real cash flows with the 1) nominal Weighted Average Cost of Capital, WACC, 2) real WACC, wacc, 3) inflated WACC, $WACC^{Inf}$ and 4) deflated WACC, $WACC^{Def}$.

The cash flows are derived from financial statements that have been constructed in nominal prices.

For consistency in valuation, we must use the deflated WACC rather than the real WACC to discount real cash flows, and the nominal WACC to discount nominal cash flows.

Keywords
Weighted Average Cost of Capital, WACC, firm valuation, capital budgeting, deflated WACC, real WACC, inflation.

JEL codes
D61, G31, H43

Introduction

In a world without taxes, the value of the deflated Weighted Average Cost of Capital, WACC applied to the real Free Cash Flow (FCF) equals the value of the real WACC applied to the real FCF. However, in a world with taxes, there is a small discrepancy between the two WACCs. This is due to the (1-T) term that is in the standard expression for the WACC applied to the Free Cash Flow (FCF). Why is this important?

For consistency in valuation, we discount the nominal cash flows with the nominal discount rate, and the corresponding real cash flows with the corresponding real discount rate, and both of these calculations must give the same present value. If not, then we know that there is a mistake in the financial model and the valuation. Since there is a discrepancy between the deflated WACC, $wacc^{Def}$, and the real WACC, wacc[1], in a world with taxes, using the wrong WACC leads to a mismatching of results that is due to the WACC rather than the financial model. Similarly, there is a difference between the inflated WACC, $WACC^{Inf}$ and the nominal WACC, WACC, as well.

The value of the discrepancy is small and the formulas for the discrepancies, as derived by Bradley and Jarrell, 2008, are as follows:

Difference in the real formulation = wacc – $WACC^{Def}$

$$= \pi \times D\% \times T/(1 + \pi) \qquad (1a)$$

Difference in the nominal formulation = $WACC^{Inf}$ - WACC

$$= \pi \times D\% \times T \qquad (1b)$$

[1] We adopt the convention that capital letters refer to nominal values and lowercase letters to real values.

Where D% is the percentage of debt, T is the tax rate and π is the expected inflation rate

From a conceptual point of view, the distinction is important, even though the differences in the valuation may be small.

Bradley and Jarrel, 2008, propose to calculate the wacc from real inputs and then inflate it to obtain the nominal WACC when using perpetuities. With this approach they obtain inflation neutral values. What we show in this work is that using the real (or inflated) WACC to discount finite real (or nominal) cash flows, creates an inconsistency in valuation, and we should use the nominal formulation.

We organize this note as follows. First, we explain the distinction between the two WACCs: $WACC^{Def}$ and wacc. Second, we provide a simple numerical example to illustrate the difference. Third, we conclude. In Appendix A, we provide the algebraic derivation of the difference.

Section One. Real, Nominal, Deflated and Inflated WACC

In the economics and financial literature, the real rate of interest is associated with the deflated rate of interest. We do recognize that a current interest rate has three components: inflation, risk and real interest rate. Hence, when we refer to the real rate we are assuming no risk premium and an inflation free rate. This real rate of interest is not observable in the economy, but it can be estimated by deflating the risk free rate, R_f with the expected inflation rate.

We use the Fisher equation in its exact multiplicative form. This is, "*the rate of interest in the (relatively) depreciating standard is equal to the sum of three terms, viz., the rate of interest in the appreciating standard, the rate of appreciation itself, and the* *product of these two elements"* (Fisher 1896, 8-9; emphasis in original, cited by Dimand, 1999, p. 746). Fisher concluded that "*The adjustment of (money) interest to long price-movements is more perfect than to short price-movements*" (1907, 283; emphasis in original). Fisher, 1930, studied this relationship and examined its statistical importance with the correlation between lagging inflation and interest rates. This is admirable given the restricted computing resources available at that time.

Then

$$1 + \text{RATE} = (1 + \text{rate})\times(1 + \pi) = 1 + \text{rate} + \pi + \pi\times\text{RATE} \tag{2a}$$

Where RATE stands for the rate in nominal terms, π is the expected inflation rate and rate stands for the real RATE.

On the other hand, it is common to use an approximation to (2a) as follows, where the nominal rate is simply the sum of the expected inflation rate and the real rate, with the assumption that the cross-term is small and may be safely ignored.

$$\text{RATE} = \pi + \text{rate} \tag{2b}$$

See for instance, the very same Fisher, 1930, Mundell, 1963, Ibrahim and Williams, 1978, Rose, 1988, Woodward, 1992, Patnaik, 2001, Choi, 2002, Perez and Siegler, 2003, Chung and Crowder, 2004, Das, 2004 and Sun and Phillips, 2004. Also, Sahu, Anandi, Jha and Meyer, 1990, use the approximation although they recognize that the correct expression is (2a). In this paper, we use the exact formulation (2a).

In the literature it is also common to consider a real interest rate as a deflated interest rate even if the nominal rate is the R_f or the return of an investment in the stock market. See for instance, Huizinga and Mishkin, 1984, Kandel, Ofer and Sarig, 1996 and Das, 2004. For us, the real interest rate comes from the deflated risk free rate; others are deflated rates.

In this work we assume the real rate is constant across borders and time. This is not strictly true, but on average it tends to be constant. This is suggested explicitly or implicitly by McMillan, Buck, and Deegan, 1984, Woodward, 1992, Kennedy, 2000, Cremers, 2001 and Chung and Crowder, 2004 when they study the real interest rate parity.

In the context of cash flow valuation, the standard textbook formula for WACC applied to the FCF is as follows:

$$WACC^{FCF} = D\% \times Kd \times (1 - T) + E\% \times Ke \tag{3a}$$

Where D% is the percentage of debt, E% is the percentage of equity, Kd is the nominal cost of debt and Ke is the nominal return to levered equity

There are two possible interpretations of the expression for the WACC in equation (3a). First, in an unrealistic world where the expected inflation rate is zero, the cost of debt and the return to equity in equation 1 are in real terms.

$$\text{Real WACC} = wacc = D\% \times kd \times (1 - T) + E\% \times ke \tag{3b}$$

Where kd is the real cost of debt and ke is the real return to levered equity

We call this wacc and distinguish this real WACC in equation (3b) from $WACC^{Def}$, as defined below.

Second, in a world with a positive expected inflation rate, the cost of debt and the return to levered equity in equation (2a) are in nominal terms.

$$\text{Nominal WACC} = WACC = D\% \times Kd \times (1 - T) + E\% \times Ke \tag{3c}$$

Where Kd is the nominal cost of debt and Ke is the nominal return to equity

In practice, we have information on the nominal values for the cost of debt and the return to equity. We do not observe real values for these parameters. Consequently, we should use the expression for the nominal WACC in equation (3c) rather than the real WACC in equation (3b). When using a financial planning model, we start from an estimation of input data such as the expected inflation rate, the real interest rate, the risk premia, etc. and use the input data to construct the nominal rates.

In the case of WACC, there is a distinction between wacc, based on parameters in real terms, and $WACC^{Def}$, which is obtained from WACC, based on parameters in nominal terms and the Fisher relationship.

$WACC^{Def}$, which is defined using the Fisher relationship, does not equal the wacc. As stated earlier, this discrepancy between the real and deflated WACCs is due to the coefficient (1 – T) that is applied to the cost of debt in the expression for the WACC in equation (3a).

$$\text{Deflated WACC} = WACC^{Def} = (WACC - \pi)/(1 + \pi) \qquad (4a)$$

In the same vein we define $WACC^{Inf}$ as

$$\text{Inflated WACC} = WACC^{Inf} = wacc \times (1 + \pi) + \pi \qquad (4b)$$

These two previous equations are based on the Fisher relationship.

To obtain $WACC^{Def}$, we subtract the expected inflation rate from the nominal WACC and divide by one plus the expected inflation rate. Using nominal values for the cost of debt and the return to equity in the expression for WACC (as in equation (3c)), and via the Fisher relationship, we obtain $WACC^{Def}$ in equation (4a). For $WACC^{Inf}$ we use the same Fisher relationship using wacc multiplied by one plus the inflation rate, plus the inflation rate.

The standard relationships between the nominal and real values for the cost of debt and the return to equity, via the Fisher relationships, are as follows:

$$1 + Kd = 1 + kd\times(1 + \pi) + \pi \ = (1+kd)\times(1 + \pi) \tag{5.1}$$

$$1 + Ke = 1 + ke\times(1 + \pi) + \pi \ = (1+ke)\times(1 + \pi) \tag{5.2}$$

When estimating the nominal or real Kd or Ke we rely on a proxy similar to the Capital Asset Pricing Model, CAPM. CAPM states that a nominal interest rate (a rate of return return) has the three above mentioned components: inflation, risk and real interest rate. The CAPM says

$$R = R_f + \beta\times(R_m - R_f) \tag{6a}$$

Where R is a nominal return, R_f is the risk free rate, R_m stands for the market return in nominal terms and $(R_m - R_f)$ stands for the market risk premium in nominal terms. The same model would be valid for "real" return, r', with only inflation excluded. For the case of r', the CAPM formulation is

$$r' = r + \beta\times(R_m - R_f)/(1+\pi) \tag{6b}$$

Where r is the real rate of interest, estimated by deflating R_f, r' is the "real" return including risk and β stands for what is known as the beta for the stock. In fact, if we inflate (6b) using the correct Fisher equation we will obtain (6a).

If one is not careful, one could easily assume (mistakenly) that the expected inflation rate should not affect the value of the WACC. However, as we show with a numerical example in Section Two, in the presence of taxes, there is an important distinction between $WACC^{Def}$ and wacc. The problem lies with the (1-T) coefficient applied to the cost of debt in the expression for the WACC, and this intuition is correct.

Let us consider the different approaches:

$$FCF_n = fcf_n \times (1+\pi)^n \tag{7}$$

Where FCF stands for the nominal free cash flow and fcf for the real free cash flow.

Case 1. Nominal FCF discounted with the WACC

$$PV(FCF_n @ \text{ WACC}) = \frac{fcf_n(1+\pi)^n}{(1+WACC)^n} = \frac{fcf_n}{\left(1+WACC^{Def}\right)^n} \tag{8}$$

Case 2. Real FCF, fcf, discounted with the wacc

$$PV(fcf_n @ wacc) = \frac{fcf_n}{(1+wacc)^n} \tag{9}$$

Case 3. Real FCF, fcf, discounted with the $WACC^{Def}$.

$$PV\left(fcf_n @ WACC^{Def}\right) = \frac{fcf_n}{\left(1+WACC^{Def}\right)^n} \tag{10}$$

Case 4. Nominal FCF, discounted with the inflated WACC, $WACC^{Inf}$.

$$PV\left(FCF_n @ \text{ WACC}^{Inf}\right) = \frac{fcf_n(1+\pi)^n}{\left(1+WACC^{Inf}\right)^n} = \frac{fcf_n}{(1+wacc)^n} \tag{11}$$

In the presence of taxes, wacc and $WACC^{Def}$ are different. Hence, (8) and (10), and (9) and (11) are respectively identical. In a world without taxes the four previous expressions are identical.

Section Two. A Simple Numerical Example

In this section, we illustrate the distinction between wacc and $WACC^{Def}$ and between WACC and $WACC^{Inf}$ with a simple numerical example.

Consider the following numerical values.

D% = 40%, E% = 60%, T = 20% and π = 5%, kd = 6% and ke = 10%,

Calculation of the real WACC with parameters in real terms

wacc = D%×kd×(1 – T) + E%×ke

= 40%×6%×(1 – 20%) + 60%×10%

$= 1.920\% + 6.000\% = 7.920\%$

For calculating WACC we introduce Ke and Kd into the equation as follows:

Using 5.1

$Kd = 6\%\times(1 + 5\%) + 5\% = \mathbf{11.300\%}$

Using 5.2

$Ke = 10\%\times(1 + 5\%) + 5\% = \mathbf{15.500\%}$

Using (4)

$WACC = D\%\times Kd\times(1 - T) + E\%\times Ke$

$= 40\%\times 11.3\%\times(1 - 20\%) + 60\%\times 15.5\%$

$= 3.616\% + 9.300\% = \mathbf{12.916\%}$

Deflated and Inflated WACC

Using the Fisher relationship (equation 4a) for $WACC^{Def}$, we obtain 7.539%, which is lower than the value of the real WACC, which is 7.92%.

$WACC^{Def} = (12.916\% - 5\%)/(1 + 5\%) = \mathbf{7.539\%}$

The difference between wacc and $WACC^{Def}$ is 0.381%.

$wacc - WACC^{Def} = 7.920\% - 7.539\% = 0.381\%$

As per equation (1a)

$D\%\times T\times\pi/(1 + \pi) = 40\% \times 20\% \times 5\%/1.05 = 0.0038095 = 0.381\%$

The $WACC^{Inf}$ is obtained using the same Fisher relationship (equation 4b) and its value is 13.160%.

$WACC^{Inf} = 7.920\% \times(1+ 5\%) + 5\% = 13.316\%$

The difference between WACC and $WACC^{Inf}$ is 0.4%, according to (1b)

$D\%\times T\times\pi = 40\%\times 20\%\times 5\% = 0.004 = 0.4\% = 13.316\% - 12.916\% = 0.4\%$

When forecasting financial statements (and deriving cash flows), we construct the financial statements from quantities and actual prices. The former are scaled with volume increases and the later with price increases that take into account inflation rates and real increases in price. In this example we assume prices are inflation neutral which means that real increases in prices are zero. Consider the following real FCF.

Table 1: Real FCF

Year	0	1	2	3	4	5
Real Free Cash Flow		257.14	254.88	254.83	250.92	250.73

With respect to year 0, the present value of the real FCF, discounted with the real WACC of 7.92% is US$ 1,016.11

In Table 2, we show the nominal FCF, which we obtained from the real FCF by inflating with the inflation index.

Table 2: Nominal FCF, US$

Year	0	1	2	3	4	5
Nominal Free Cash Flow (inflated)		270.00	281.00	295.00	305.00	320.00

With respect to the end of year 0, the present value of the nominal FCF, discounted with WACC of 12.916% is US$ 1,026.36. This is the correct value given that the model intends to forecast what is going to happen in the future. Hence, other methods that depart from this value are considered incorrect and inconsistent.

Thus, we can see that the nominal FCF, discounted with the nominal WACC gives a value that is higher than the real FCF, discounted with the real WACC.

It is easy to verify that, with respect to year 0, the real FCF, discounted with $WACC^{Def}$ of 7.539% gives the same present value as the nominal FCF discounted with the nominal WACC. This is shown in table 3.

In table 3 we show the present values of the two FCF (real and nominal) discounted with WACC, wacc, $WACC^{Def}$ and $WACC^{Inf}$.

Table 3. Present Value of Real and Nominal FCF at Different Inflation Rates

Inflation rate	PV nominal CF at WACC	PV real CF at $WACC^{Def}$	PV real CF at wacc	PV nominal CF $WACC^{Inf}$
0.0%	1,016.11	1,016.11	1,016.11	1,016.11
2.5%	1,021.34	1,021.34	1,016.11	1,016.11
5.0%	1,026.36	1,026.36	1,016.11	1,016.11
7.5%	1,031.19	1,031.19	1,016.11	1,016.11
10.0%	1,035.83	1,035.83	1,016.11	1,016.11
12.5%	1,040.29	1,040.29	1,016.11	1,016.11
15.0%	1,044.59	1,044.59	1,016.11	1,016.11

Observe that the PV for nominal FCF at nominal WACC (Column 1) and real FCF at $WACC^{Def}$ (Column 2) are identical and non neutral to inflation; also observe that they are consistent as they should be. The present value of the real FCF at wacc (Column 3) and the nominal cash flow at $WACC^{Inf}$ (Column 4) are identical as expected and are inflation neutral.

The table shows that inflation creates value and this may appear to be strange; the higher the expected inflation rate, the higher is the PV. However, this is consistent because the higher expected inflation rate means that the present value of the interest payments is higher and this in turn means that the present of the tax shields is higher. As can be seen in table 3, neither PV of real cash flows at wacc (Column 3) nor PV of nominal cash flows at $WACC^{Inf}$ (Column 4) are consistent with the PV of nominal cash flows at WACC (Column 2).

In the next table we show the same prevent values of the cash flows without taxes. In table 4 we show the present values of the two FCF (real and nominal) discounted with WACC, wacc, $WACC^{Def}$ and $WACC^{Inf}$ with no taxes.

Table 4. Present Value of Real and Nominal FCF at Different Inflation Rates, no Taxes

	PV nominal CF at WACC	PV real CF at wacc	PV real CF at $WACC^{Def}$	PV nominal CF at $WACC^{Inf}$
0.0%	1,003.43	1,003.43	1,003.43	1,003.43
2.5%	1,003.43	1,003.43	1,003.43	1,003.43
5.0%	1,003.43	1,003.43	1,003.43	1,003.43
7.5%	1,003.43	1,003.43	1,003.43	1,003.43
10.0%	1,003.43	1,003.43	1,003.43	1,003.43
12.5%	1,003.43	1,003.43	1,003.43	1,003.43
15.0%	1,003.43	1,003.43	1,003.43	1,003.43

In table 4, we find consistency between all present values at any inflation rate, as predicted: all are identical.

As shown in Vélez-Pareja, 2006, under certain conditions, using real or constant prices overvalue the cash flow appraisal. The conditions, among others, include the existence of taxes, depreciation and accounts receivable. In this simple example we have assumed no depreciation and no accounts receivable, nor payable.

Conclusion

In this note, using a simple numerical example, we have shown that in a world with taxes, there is a discrepancy between $WACC^{Def}$ and wacc, and between WACC and $WACC^{Inf}$. This means that under the restricted conditions of no depreciation, no accounts receivable and payable, it is equivalent and correct to value the nominal cash flows at the nominal WACC and the real cash flows at $WACC^{Def}$. Correspondingly, it is wrong to value the real cash flows at wacc and the nominal cash flows at $WACC^{Inf}$.

For consistency in valuation, we must use $WACC^{Def}$ rather than wacc in discounting real free cash flows, as proposed by Bradley and Jarrell.

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APPENDIX A

The expression for the nominal WACC is as follows:

$$\text{WACC} = \text{D\%}\times\text{Kd}\times(1 - \text{T}) + \text{E\%}\times\text{Ke} \qquad \text{(A1)}$$

Substituting equations 5.1 and 5.2 into the right side of equation A1, we obtain,

$$\begin{aligned}&\text{WACC}\\&= \text{D\%}\times[\text{kd}\times(1 + \pi) + \pi]\times(1 - \text{T}) + \text{E\%}\times[\text{ke}\times(1 + \pi) + \pi] \qquad \text{(A2)}\end{aligned}$$

Next, using the Fisher relationship in equation 4a, we can rewrite the left side of equation A1 in terms of the deflated WACC as follows.

$$\begin{aligned}&\text{WACC}^{\text{Def}}\times(1 + \pi) + \pi\\&= \text{D\%}\times[\text{kd}\times(1 + \pi) + \pi]\times(1 - \text{T}) + \text{E\%}\times[\text{ke}\times(1 + \pi) + \pi] \qquad \text{(A3)}\end{aligned}$$

Simplifying, we obtain,

$$\begin{aligned}\text{WACC}^{\text{Def}}\times(1 + \pi) + \pi = {}& \text{D\%}\times\text{kd}\times(1 + \pi)\times(1 - \text{T}) + \text{D\%}\times\pi\times(1 - \text{T})\\&+ \text{E\%}\times\text{ke}\times(1 + \pi) + \text{E\%}\times\pi \qquad \text{(A4)}\end{aligned}$$

Rearranging, we obtain,

$$\begin{aligned}\text{WACC}^{\text{Def}}\times(1 + \pi) + \pi = {}& \text{D\%}\times\text{kd}\times(1 + \pi)\times(1 - \text{T}) + \text{D\%}\times\pi\\&- \text{D\%}\times\pi\times\text{T} + \text{E\%}\times\text{ke}\times(1 + \pi) + \text{E\%}\times\pi \qquad \text{(A5.1)}\end{aligned}$$

As we know, D% plus E% is 100%, then

$$\begin{aligned}\text{WACC}^{\text{Def}}\times(1 + \pi) = {}& \text{D\%}\times\text{kd}\times(1 + \pi)\times(1 - \text{T})\\&+ \text{E\%}\times\text{ke}\times(1 + \pi) - \text{D\%}\times\pi\times\text{T} \qquad \text{(A5.2)}\end{aligned}$$

$$\text{WACC}^{\text{Def}} = \text{D\%}\times\text{kd}\times(1 - \text{T}) + \text{E\%}\times\text{ke} - \text{D\%}\times\pi\times\text{T}/(1 + \pi) \qquad \text{(A5.3)}$$

Compare equation (3b) with equation A5.3. The extra term in A5.3 is the expression for the difference between wacc and WACC^{Def}.

The other way around, if we begin with the wacc, and inflate it to $WACC^{Inf}$ we have:

$$wacc= D\%\times kd\times(1 – T) + E\%\times ke \qquad (A6)$$

Replacing kd and ke from (5.1) and (5.2) in (A6) we have

$$wacc= D\%\times(Kd-\pi)\times(1 – T)/(1 + \pi) + E\%\times(Ke-\pi)/(1 + \pi) \qquad (A7a)$$

$$wacc\times(1 + \pi)= D\%\times(Kd-\pi)\times(1 – T) + E\%\times(Ke-\pi) \qquad (A7b)$$

$$wacc\times(1 + \pi) = D\%\times Kd\times(1 – T) + E\%\times Ke -\pi\times[D\%\times(1 – T) + E\%] \qquad (A7c)$$

$$wacc\times(1 + \pi) = WACC -\pi\times[1 – D\%\times T] \qquad (A7d)$$

$$wacc\times(1 + \pi) = WACC -\pi + \pi\times D\%\times T] \qquad (A7e)$$

$$wacc\times(1 + \pi) + \pi = WACC + \pi\times D\%\times T \qquad (A7f)$$

But

$$WACC^{Infl} = wacc\times(1 + \pi) + \pi \qquad (A8)$$

Hence, there is a difference between $WACC^{Infl}$ and WACC.

$$WACC^{Infl} - WACC = \pi\times D\%\times T \qquad (A9)$$