Calculating Betas

Ignacio Vélez-Pareja
ivelez@unitecnologica.edu.co
nachovelez@gmail.com
Universidad Tecnológica de Bolívar
Department of Finance and International Business
Instituto de Estudios para el Desarrollo (IDE)
Cartagena, Colombia

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Abstract

This teaching note shows the relationship between levered and unlevered betas and the general formulation for the cost of equity. It also shows, step by step, the procedure to estimate betas from data found in the stock market.

It shows well known procedures for estimating betas: correlation coefficient and standard deviations of the stock and the market, covariance between stock and market returns and market variance and finally, ordinary least squares (numerical and graphical).

This written material is useful for practitioners, teachers and students of Corporate Finance.

Keywords
Betas, beta calculation, stock returns, market returns, systematic risk.

JEL Classifications

G10, G11, G12

Introduction

When defining the cost of capital it is necessary to estimate the cost of equity. This cost of equity has to be calculated taking into account the risk perceived by shareholders on the firm and it is associated to systematic risk. Systematic risk is measured by the beta coefficient or simply, by the beta of the stock.

The risk associated to the firm has two components: systematic and non systematic risk. Systematic risk depends on general conditions associated to an economy as a whole or to an industry. It is common to a group of firms and is not avoidable. Systematic risk can be estimated by the betas of each stock. Betas are a measure of how the systematic risk is related to the general risk or market risk.

Market risk is measured with indexes that keep track of the average price of stocks that compose the index.

Beta is measured relating stock risk with market risk. This is shown in equations (1) and (2) as follows:

$$\beta_s = \sigma_s cor(Rm,Rs)/\sigma_m \qquad (1a)$$

This formula says that beta is calculated as the product of the standard deviation of the stock, σ_s and the correlation between the stock return and market return, cor(Rm,Rs), divided by the standard deviation of the market return, σ_m.

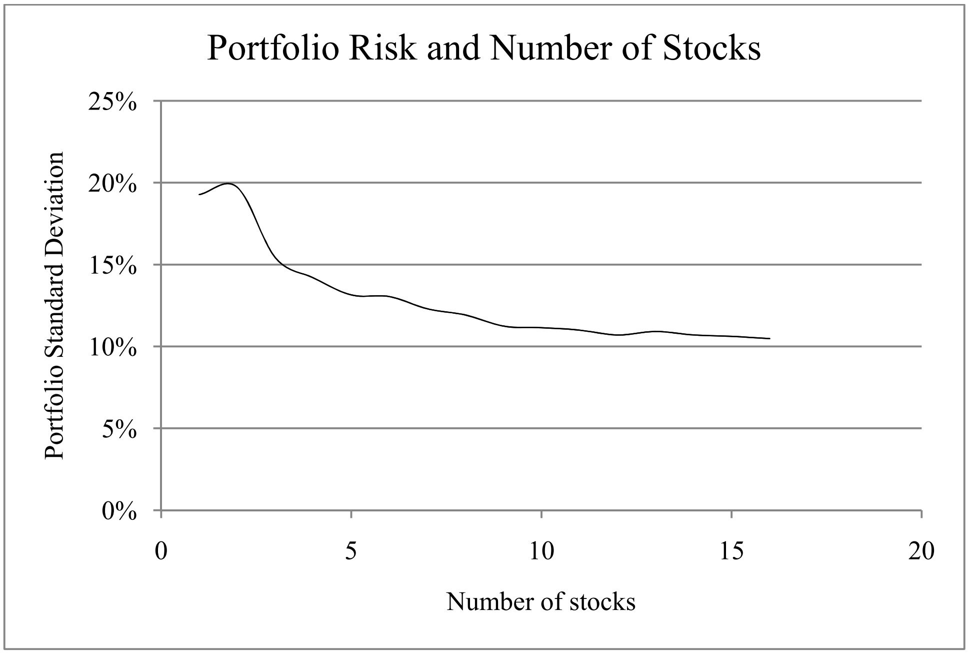
$$\beta = cov(Rm,Rs)/\sigma_m^2 \qquad (1b)$$

This second version for calculating beta says that beta is the covariance between the stock return and market return, cov(Rm,Rs) divided by the variance of market return, σ_m^2.

Non systematic risk is idiosyncratic and is specific to a firm. This type of risk is avoidable through diversification.

The idea of diversification is very old. An old English proverb says: “Don't put all your eggs in one basket”. Using this idea, naïve portfolios were designed with n = 1, 2, 3, up to 16 stocks where the proportion of each stock was identical to 1/n. The portfolios were constructed starting from an alphabetical list of stocks that means they were selected at random. Exhibit 1 shows the risk (standard deviation of portfolio return) and the number of stocks in the portfolio.

Exhibit 1. Portfolio risk and Diversification



As can be seen, there is a trend to stabilize the risk of the portfolio at nearly 10%. This might be an estimation of the systematic risk that cannot be eliminated through diversification.

This teaching note distinguishes between the levered cost of equity, Ke and the unlevered cost of equity, Ku. The first one is the expected return by shareholders from a firm with a given level of debt or leverage. The observed stock return is compared with the expected return, Ke.

There is a non-observable return called the cost of unlevered equity, Ku and is the return a stockholder would expect if the firm has no debt at all.

Each return has associated a beta. This beta is an integral part of a popular model known as CAPM for Capital Asset Pricing Model. The CAPM is expressed as

$$Rs = R_f + \beta ERP = R_f + \beta[E(Rm)-R_f] \quad (2)$$

Where Rs is the stock return, R_f is the risk free rate, β is the risk of the stock or beta and ERP is the equity risk premium, also known as MRP, market risk premium and is the difference between the expected market return, E(Rm) and the risk free rate, R_f.

The CAPM can be used to define Ke, Ku and cost of debt, Kd.

Next section discusses the relationship between the levered beta and the unlevered beta. Second section shows the procedure to estimate a beta. Third section summarizes this teaching note.

The Relationship Between the Levered Beta and the Unlevered Beta

This section discusses the relationship between the unlevered beta and the levered beta.

Tham and Velez-Pareja (2004) show that the general expression for the return to levered equity Ke_i is

$$Ke_i = Ku_i + (Ku_i - Kd_i)\frac{D_{i-1}}{E^L_{i-1}} - (Ku_i - \psi_i)\frac{V^{TS}_{i-1}}{E^L_{i-1}} \qquad (3)$$

Assume that the return to unlevered equity Ku_i, the cost of debt Kd_i and the discount rate for the tax shield ψ_i are constant. Ke is the expected cost of levered equity, V^{TS} is the value of the tax shields, TS and E^L is the value of levered equity.

Ke_i is a function of the debt-equity ratio and the ratio of the value of the tax shield to the value of the levered equity. If at least one of these two ratios changes over time, the return to levered equity Ke_i will change.

Using CAPM, the expressions for the cost of debt Kd_i, the return to unlevered equity Ku_i and the return to levered equity Ke_i can be written as follows.

$$Kd_i = R_f + \beta_D[E(R_m) - R_f] \qquad (4)$$

$$Ku_i = R_f + \beta_U[E(R_m) - R_f] \qquad (5)$$

$$Ke_i = R_f + \beta_L[E(R_m) - R_f] \qquad (6)$$

β_L is the beta for the levered firm and in the return to levered equity in equation 5, it is affected by a specified leverage; β_U is the beta for the unlevered return and β_D is the beta for debt.

If you assume that ψ_i is equal to Ku_i, then equation 3 collapses to (7a) as follows.

$$Ke_i = Ku_i + (Ku_i - Kd_i)D_{i-1}/E^L_{i-1} \qquad (7a)$$

When ψ_i is equal to Kd_i and the cash flows are a perpetuity,

$$Ke = Ku_i + (Ku_i - Kd_i)\frac{D_{i-1}}{E^L_{i-1}} - (Ku_i - Kd_i)\frac{TD_{i-1}}{E^L_{i-1}}$$
$$= Ku_i + (Ku_i - Kd_i)(1-T)\frac{D_{i-1}}{E^L_{i-1}} \tag{7b}$$

T is the corporate tax rate.

When ψ_i is equal to Ke_i, Tham, Velez-Pareja and Kolari (2010) and Kolari and Vélez-Pareja (2010) have shown that

$$Ke_i = Ku_i + \frac{(Ku_i - Kd_i)\times D_{i-1}}{V^{Un}_{i-1} - D_{i-1}} \tag{7c}$$

It is important to be clear about the reasons for the different expressions for the return to levered equity in equations 7a, 7b and 7c. Expression 7b is valid only for cash flows in perpetuity and assumes that the discount rate for the tax shield is equal to the cost of debt, Kd. Expression 7a holds for both finite cash flows and perpetuities and assumes that the discount rate for the tax shield is equal to the return to unlevered equity, Ku. Expression 7c holds for both, perpetuities and finite cash flows and assumes Ke as the discount rate for tax shields.

The typical work is with finite cash flows and assumes that the discount rate for the tax shield is the cost of debt hence formulas 7b cannot be used. For finite cash flows, you use the following expression.

$$Ke = Ku_i + (Ku_i - Kd_i)\left[\frac{D_{i-1}}{E^L_{i-1}} - \frac{V^{TS}_{i-1}}{E^L_{i-1}}\right] \tag{8}$$

For the case of ψ_i = Ku, and substituting equations 4 and 5 into equation 7a and simplifying, you obtain, as Tham and Velez-Pareja (2004) show

$$Ke_i = R_f + \beta_U[E(R_m) - R_f] + (\beta_U - \beta_D)[E(R_m) - R_f]D_{i-1}/E^L_{i-1}$$
$$= R_f + [\beta_U + (\beta_U - \beta_D)D_{i-1}/E^L_{i-1}][E(R_m) - R_f] \tag{9}$$

Now compare equations 6 and 9: there you find the relationship between the beta for levered equity β_L and the unlevered beta, β_U.

$$\beta_L = \beta_U + (\beta_U - \beta_D)D_{i-1}/E^L_{i-1} = \beta_U(1 + D_{i-1}/E^L_{i-1}) - \beta_D D_{i-1}/E^L_{i-1} \tag{10}$$

Now, for the case ψ_i = Kd, substituting equations 4 and 5 into equation 7b and simplifying, you obtain,

$$Ke = R_f + \left[\beta_U + (\beta_U - \beta_D)(1-T)\frac{D_{i-1}}{E^L_{i-1}}\right][E(Rm) - Rf] \qquad (11)$$

Comparing with (6) you find the relationship between levered and unlevered beta.

$$\beta_L = \beta_U + (\beta_U - \beta_D)(1-T)\frac{D_{i-1}}{E^L_{i-1}} \qquad (12a)$$

$$\beta_L = \beta_U\left[1 + (1-T)\frac{D_{i-1}}{E^L_{i-1}}\right] - \beta_D(1-T)\frac{D_{i-1}}{E^L_{i-1}} \qquad (12b)$$

In the same vein, when ψ_i is equal to Ke_i

$$\beta_L = \beta_U + (\beta_U - \beta_D)D_{i-1}/(V_{Un} - D_{i-1}) \qquad (13a)$$

$$\beta_L = \beta_U + \beta_U D_{i-1}/(V_{Un} - D_{i-1}) - \beta_D D_{i-1}/(V_{Un} - D_{i-1}) \qquad (13b)$$

Assuming β_D equal to 0, which means a private debt (from a bank, for instance) that is not traded at the stock market, and then you have:

When ψ_i = Ku

$$\beta_L = \beta_U(1 + D_{i-1}/E^L_{i-1}) \qquad (14a)$$

When ψ_i = Kd and cash flows are perpetuities

$$\beta_L = \beta_U\left[1 + (1-T)\frac{D_{i-1}}{E^L_{i-1}}\right] \qquad (14b)$$

And finally, when ψ_i = Ke.

$$\beta_L = \beta_U(1 + D_{i-1}/(V_{Un}-D)) \qquad (14c)$$

Now, from equations 14a to 14c solve for β_U.

If you assume that the discount rate for tax savings is Ku, then

$$\beta_{Ui-1} = \frac{\beta_L}{\left[1 + \frac{D_{i-1}}{E_{i-1}}\right]} \qquad (15a)$$

When you assume ψ = Kd

$$\beta_{Ui-1} = \frac{\beta_L}{\left[1 + \frac{D_{i-1}}{E_{i-1}}(1-T)\right]} \qquad (15b)$$

If you assume Ke as the discount rate for TS, then

$$\beta_{Ui-1} = \frac{\beta_L}{\left[1 + \frac{D_{i-1}}{V_U - D_{i-1}}\right]} \qquad (15c)$$

Where V_U is the unlevered value of the firm.

Applying 15a to 15c is known as unlevering betas.

Estimating Beta

There are many sources to collect data for estimating betas. One of them is the official site for the stock market, for instance, The New York Stock Exchange, NYSE, or Nasdaq, and the respective office in any country. Also there other non official sources such as Yahoo! Finance, Google Finance, Bloomberg, DataValue, Compustat, Economatica, etc. As a last resource, use the firm website.

Table 1 shows a small sample of stock prices and indexes for 26 periods. The market return is calculated as

$$R_{mt}=\frac{Index_t}{Index_{t-1}}-1 \qquad (16a)$$

For year 1 you have:

$$R_{m1}=\frac{941.88}{965.46}-1= -2.4\%$$

The stock return is calculated as

$$R_{st}=\frac{P_t}{P_{t-1}}-1 \qquad (16b)^1$$

For year 1 you have

$$R_{s1}=\frac{22,111.05}{21,891.25}-1=1.0\%$$

Repeating these calculations you get table 1.

[1] Strictly, $R_{st}=\frac{P_t+Div_t}{P_{t-1}}-1$ where Div is dividends paid to shareholders.

Table 1. Monthly Prices, Index Values and Returns

Month	Index	Index return	Stock price	Stock return
0	965.46		21,891.25	
1	941.88	-2.4%	22,111.05	1.0%
2	861.05	-8.6%	20,587.35	-6.9%
3	798.86	-7.2%	19,219.67	-6.6%
4	786.63	-1.5%	19,202.71	-0.1%
5	825.29	4.9%	19,490.81	1.5%
6	880.91	6.7%	19,845.01	1.8%
7	801.48	-9.0%	18,259.85	-8.0%
8	783.49	-2.2%	17,905.40	-1.9%
9	725.81	-7.4%	17,410.63	-2.8%
10	682.58	-6.0%	17,457.45	0.3%
11	717.41	5.1%	18,163.65	4.0%
12	748.66	4.4%	18,391.51	1.3%
13	812.81	8.6%	18,609.25	1.2%
14	776.63	-4.5%	18,313.36	-1.6%
15	801.94	3.3%	18,347.76	0.2%
16	874.70	9.1%	19,698.04	7.4%
17	870.07	-0.5%	19,485.54	-1.1%
18	848.24	-2.5%	18,576.20	-4.7%
19	854.67	0.8%	18,074.39	-2.7%
20	862.90	1.0%	18,317.12	1.3%
21	872.02	1.1%	18,499.30	1.0%
22	848.18	-2.7%	18,500.00	0.0%
23	838.20	-1.2%	18,496.65	0.0%
24	958.79	14.4%	20,780.78	12.3%
25	1,037.34	8.2%	21,275.99	2.4%
26	1,068.31	3.0%	22,618.72	6.3%

Using the statistics from market and stock data such as standard deviations for 1) market and 2) stock, 3) correlation coefficient between market and stock return you obtain 4) beta as in table 2:

Table 2. Statistics from data

Market Return Standard Deviation, σ_m	0.0589586
Stock Return Standard Deviation, σ_s	0.0429376
Correlation coefficient Market and Stock Return, cor(Rm,Rs)	0.8442786
$\beta=\sigma_s cor(Rm,Rs)/ \sigma_m$	0.6148606

Calculating and using 1) covariance between market and stock returns and 2) market return variance you obtain 3) beta for the stock as in table 3.

Table 3. Calculating Beta with Statistics from Stock and Market

cov(Rm,Rs)	0.002137
Market Return Variance, σ_m^2	0.00347612
β=cov(Rm,Rs)/σ_m^2	0.6148606

Another option to estimate beta is to run an Ordinary Least Squares[2] regression between market return, Rm and stock return, Rs, as follows in tables 4 and 5:

Table 4. Using Ordinary Least Squares, OLS

Summary Output	
Regression Statistics	
Multiple R	0.84428
R Square	0.71281
Adjusted R Square	0.70084
Standard Error	0.02395
Observations	26

Table 5. ANOVA Table

	df	SS	MS	F	Significance F
Regression	1	0.03417	0.03417	59.5673	5.9E-08
Residual	24	0.01377	0.00057		
Total	25	0.04793			
	Coefficients	Standard Error	t Stat	P-value	
Intercept	-0.0013	0.00472	-0.27229	0.78773	
Index return, β_L	0.6148606	0.07967	7.71798	5.9E-08	

As can be seen all methods estimate β as 0.6148606. The ANOVA table says that β is statistically significant (p-value = 5.9E-08), the intercept is not statistically significant and could be assumed as zero (as expected, p-value = 0.78773) and the

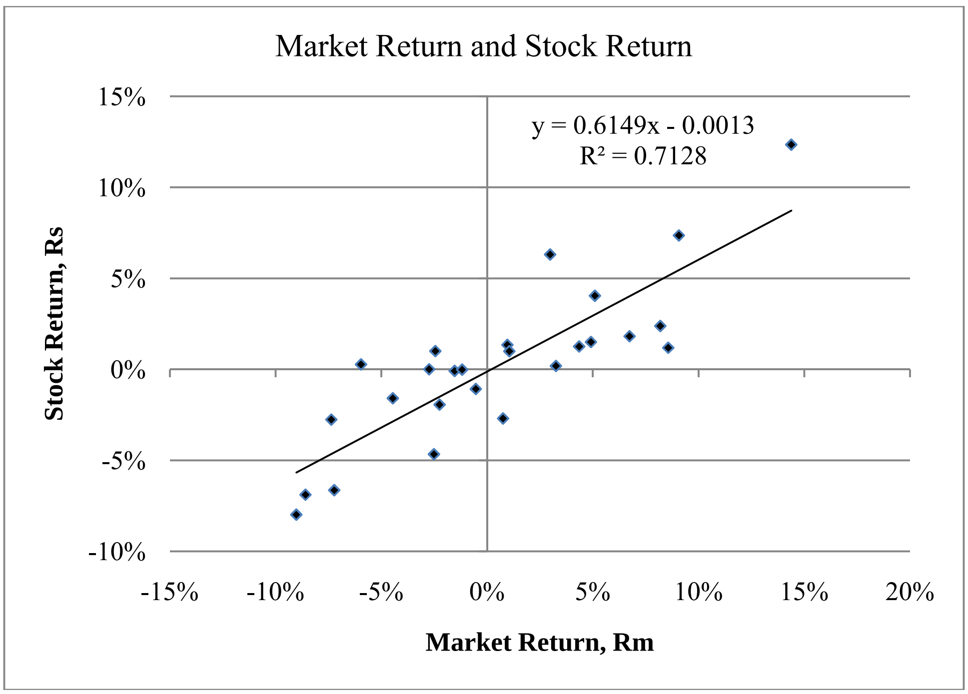
[2] This can be done with any version of Excel®. You have to activate Add-ins and the option Data Analysis will appear.

model is statistically significant as well (significance of F = 5.9E-08). The Rm explains 70.084% of the variability of Rs.

With this beta you can estimate Ke for the stock. If you wish to use the estimate of β for a non traded firm, you have to unlever betas collecting data for the industry and average them weighted by market capitalization. Once you have the average for the unlevered beta you can use CAPM to estimate Ku and use equations 7a, 7b or 7c to estimate Ke. Equations 7a and 7b create circularity. Equation 7c is free from circularity.

Exhibit 2 shows how the data looks and the lineal trend line[3]. The coefficient for the Index or Market return is indicated in the graph. Beta is that coefficient that measures the slope of the line.

Exhibit 2. Market and Stock Return with Trend Line



A continuación un ejemplo hipotético de cómo proceder para el cálculo de las betas desapalancadas. Se tienen empresas del mismo sector con los siguientes datos

[3] This can be done with any version of Excel®.

de valores de deuda, de patrimonio (valor de mercado), de valor ahorros en impuestos, V^{TS}, de tasa de impuestos y de betas calculadas, así:

See this hypothetical example on how to proceed with the calculation of the unlevered betas. It has four firms in the same industry with the following data for debt, equity (market value), value of tax savings, V^{TS}, tax rate and levered betas as follows:

Table 6. Example on how to unlever betas. Input data

Firm	D_{t-1}	E_{t-1}	V^{TS}_{t-1}	β_L	Weight by equity
A	180	300	12	1.2	0.250
B	250	350	14	1.4	0.292
C	200	300	13	1.3	0.250
D	250	250	20	1.6	0.208

When ψ = Kd unlevered betas are (using (15b)):

Table 7. Unlevering betas assuming ψ = Kd

Firm	β_L	T	D/E	β_U	β_U weighted
A	1.2	30.00%	60.00%	0.84507	0.211268
B	1.4	30.00%	71.40%	0.933458	0.272259
C	1.3	30.00%	66.70%	0.886223	0.221556
D	1.6	30.00%	100.00%	0.941176	0.196078
			β_U weighted.		0.90116
			β_U not weighted.		0.9015

When ψ = Ku unlevered betas are (using (15a)):

Table 8. Unlevering betas assuming ψ = Ku

Firm	$Beta^L$	D/E	β_U	β_U weighted
A	1.2	60.00%	0.7500	0.1875
B	1.4	71.40%	0.8168	0.2382
C	1.3	66.70%	0.7798	0.1950
D	1.6	100.00%	0.8000	0.1667
		β_U weighted		0.7874
		β_U not weighted		0.7867

When ψ = Ke unlevered betas are (using (15c)):

Table 9. Unlevering betas assuming ψ = Ke

Firm	$Beta^L$	D/(Vun-D)	β_U	β_U Ponderada
A	1.2	62.50%	0.7385	0.1846
B	1.4	74.40%	0.8027	0.2341
C	1.3	69.69%	0.7661	0.1915
D	1.6	108.70%	0.7667	0.1597
		β_U ponderada.		0.7700
		β_U no ponderada.		0.7685

Betas are weighted with the market value of equity. Now with unlevered beta, Ku can be calculated using (6) and the respective Ke with the appropriate formula (7a) (7b) or (7c). There is the question of whether or not Ku should depend on the assumption that is made on the discount rate of tax shields.

Summary

This teaching note has presented the relationship between levered and unlevered betas and the procedure to derive the later from the former that is the one you can observe from the market. With the unlevered beta you can use CAPM to estimate Ku and from that you can estimate Ke using the formulas provided.

The teaching note has also shown three procedures to estimate betas: two of them use the market and stock statistics and the third uses OLS to find the levered beta for a traded firm. The three methods give the same results as expected.

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